EXHIBIT 5

DECLARATION OF RONNIE ELLIOTT

Ronnie Elliott, being first duly sworn upon her oath, deposes and says:

1. I am over the age of eighteen and am competent to testify as to all the matters set forth herein and would so testify if called upon to do so.

2. I have personal knowledge of the matters set forth herein. My personal knowledge is based upon my observations and personal participation in the events described below.

3. I was employed with Maricopa County Community College District ("MCCCD") from the early 1990s through January 2010. I served as Phoenix College's ("PC") Vice President of Administrative Services from approximately 2007 through January 2010. On January 30, 2010 I voluntarily retired after an approximately 20 year long career with MCCCD. Following my retirement, Paul DeRose took over my responsibilities as PC's Vice President of Administrative Services. I now reside in Buffalo, Missouri.

4. In my former role as PC Vice President of Administrative Services, I was responsible for supervising our various vendor services including custodians, information technology ("IT") personnel, and copy service personnel. I also met and interacted with numerous faculty members. One such faculty member was Math Professor Dr. Cleopatria Martinez.

5. Although I was not involved with the PC's IT department's day-to-day tasks, I oversaw the IT Department's general operations and computer maintenance. Specifically, the IT Department is responsible for maintaining PC's computer systems and ensuring: (1) that no unauthorized software has been downloaded that could damage PC computers; and (2) that PC systems are not being accessed by unauthorized individuals. In my former role as Vice President of Administrative Services, the IT Department reported to me and any suspicious findings were forwarded to my attention.

6. In or around Fall of 2009, while the IT department was performing routine computer maintaince, it was discovered that an illegal file sharing program called LimeWire had been installed on Dr. Martinez's laptop computer. It was further discovered that a high school level homework assignment authored by an individual with the last name "Martinez" was present on Dr. Martinez's MCCCD issued laptop. In accordance with the IT Department's normal practice, the findings were immediately forwarded to my attention. Upon my review of the findings, I concluded that Dr. Martinez had improperly permitted an unauthorized user to access her MCCCD laptop computer. Since I was not responsible for disciplining faculty members, I notified Vice President of Academic Affairs Cassandra Kakar and PC President Anna Solley of the findings.

7. A few weeks later, on or around September 24, 2009, I learned that PC custodian Verna Daubney, had found dozens of manila folders containing confidential faculty applicant information (including social security numbers and school transcripts) in Dr. Martinez's office garbage can. Dr. Martinez's disposal of these materials in this way violated MCCCD's policies and procedures which require that applicant materials be disposed of in a secure manner. I reported the findings to PC's Vice President of Academic Affairs, Casandra Kakar, and PC President Anna Solley.

8. In January of 2010, our copy service provider, IKON, notified me that that Dr. Martinez had submitted a set of suspicious copy requests. Apparently, Dr. Martinez asked IKON to copy approximately 30 sets of a 73 page document titled "2009 MAT 182." Dr. Martinez had also submitted a similar request for her MAT 187 course. IKON was concerned that the documents appeared to contain math problems taken directly from textbooks and wanted my guidance regarding whether to process the print jobs. At my request, IKON forwarded a copy of the "2009 MAT 182" document to my attention. A true and correct copy of these materials are attached as Exhibit A.

9. After reviewing the materials I received from IKON, I concluded that the materials presented a serious risk of copyright infringement and forwarded the materials to PC Vice President of Academic Affairs, Cassandra Kakar, and PC President Anna Solley. Drs. Kakar and Solley reached the same conclusion and instructed me to bring the matter to the attention of Dr. Martinez. Accordingly, on or around January 12, 2010, I sent an email to Dr. Martinez explaining that PC had "red flagged" items Dr. Martinez had submitted for printing for the Fall 2009 and Spring 2010 semesters and that we were concerned that the materials may subject PC to a claim of copyright infringement. A true and correct copy of my January 12, 2010 email to Dr. Martinez is attached as Exhibit B.

10. Much to my surprise, rather than apologize for her behavior and remove any copyright portions of her materials, Dr. Martinez began asking me (and others within MCCCD's administration) if she could "fix" the copyright issues by changing one or two numbers from problems that were copied directly from copyright protected textbooks. Dr. Martinez's attempts to elicit advice from me (and others) on how to evade applicable copyright laws made me feel extremely uncomfortable. It also became quite apparent to me that Dr. Martinez had no intention of abiding by copyright laws and regulations.

11. On January 26, 2010, I sent Dr. Martinez another email outlining MCCCD's concerns regarding her suspected copyright violations. Specifically, I explained that I had solicited the advice of district counsel, Margaret McConnell regarding copyright infringement and fair use. I further explained that we were concerned that Dr. Martinez's actions could subject MCCCD to a copyright infringement claim because we were able to identify at least 10 instances where problems had been copied directly from the Sullivan and Sullivan Precalculs textbook. A true and correct copy of my January 26, 2010 email to Dr. Martinez is attached as Exhibit C. I retired from MCCCD several days later on January 30, 2010.

12. My decisions to report Dr. Martinez's above referenced actions to Drs. Solley and Kakar were not based on any discriminatory or retaliatory feelings towards

Dr. Martinez. To the contrary, I felt obligated to report my concerns regarding Dr. Martinez to PC administration in light of the potential legal liability that PC could have faced as a result of Dr. Martinez's actions.

Pursuant to the laws of the State of Arizona, I declare under penalty of perjury that the foregoing is true and correct.

Executed this 9th day of October, 2013 in Buffalo, Missouri.

Ronnie Elliott

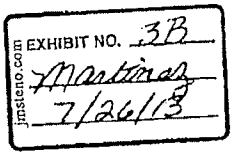
RONNIE ELLIOTT

Exhibit A

Phoenix College
Fall 2009

MAT 182
Precalculus

Trigonometry Section
Lecture Notes



Dr. Cleopatria Martínez

Chapter 1 – Trigonometry

1.1 Angles and their Measure , Basic Angles, Degrees° Minutes' Seconds", Degrees, Radians, arc length, area of a sector, linear speed, angular speed

Vocabulary: Angle, ray, line, segment, standard position of an angle, initial side,

A segment is a set of points with a starting point and an ending point.

A line is a set of points that go forever in two directions.

A ray is a set of points that start at one point then go forever.

An angle is two rays with the same starting point.

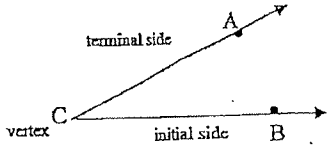
The vertex is the point at the corner of the angle.

The initial side is the ray where the angle starts.

The terminal side is the ray where the angle ends.

A positive angle opens counter-clockwise.

A negative angle opens clockwise.



The name of the angle can be the vertex, like angle C i.e. $\angle C$ or by using three letters with the vertex in the middle i.e. $\angle ACB$ or $\angle BCA$.

Standard position has the vertex at the origin and the initial side on the positive x-axis.

The size of an angle is measured by degrees. A full rotation is 360° *i.e. 360 degrees.*

Degrees° Minutes' Seconds" are used to describe the size of an angle.

One degree, written 1°, *represents* $\frac{1}{360}$ of a full rotation.

One minute, written 1', represents $\frac{1}{60}$ of a degree.

A degree can be divided into 60 parts called 60 minutes i.e. 60'.

One second written 1", represents $\frac{1}{60}$ of a minute.

A degree can be divided into 60 parts called 60 seconds i.e. 60".

An acute angle has between 0 degrees and 90 degrees.

A right angle has exactly 90 degrees.

An obtuse angle has between 90 degrees and 180 degrees.

A straight angle has exactly 180 degrees.

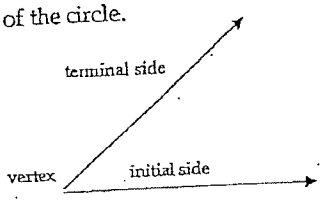
A reflexive angle has between 180 degrees and 360 degrees.

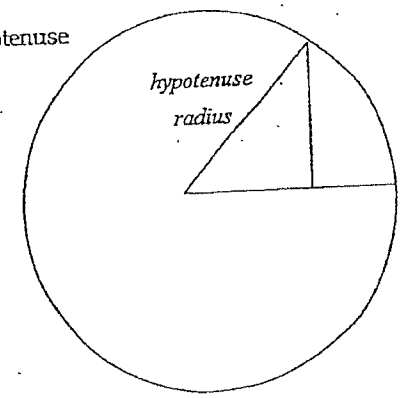
A complete rotation gives an angle measuring 360 degrees.

Complementary angles are two angles which add up to 90 degrees.

Supplementary angles are two angles which add up to 180 degrees.

Note the right triangle inscribed in the circle.
The terminal side of the angle is not only the hypotenuse of the right triangle but it is also the radius of the circle.



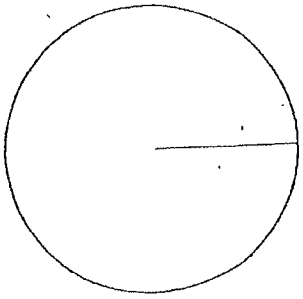


1 degree$^{\circ}$ = 60 minutes = 59 minutes + 60 seconds

1 minute$'$ = $1' = \left(\frac{1}{60}\right)^{\circ}$

1 second$''$ = $1'' = \left(\frac{1}{60}\right)' = \left(\frac{1}{60} \cdot \frac{1}{60}\right)^{\circ}$

1 minute$'$ = 60 seconds



Change *Degrees-Minutes-Seconds* into *Degrees* in decimal form.
Give the decimal answer accurate to two decimal places.

Example: Change 50°6′ 21″ into degrees.

Answer:

$50^{\circ} + 6' + 21''$

$50^{\circ} + 6\left(\frac{1}{60}\right)^{\circ} + 21\left(\frac{1}{60}\right)'$

$50^{\circ} + \frac{6}{1}\left(\frac{1}{60}\right)^{\circ} + \frac{21}{1}\left(\frac{1}{60} \cdot \frac{1}{60}\right)^{\circ}$

$50^{\circ} + \left(\frac{1}{10}\right)^{\circ} + \frac{3 \cdot 7}{1}\left(\frac{1}{3 \cdot 20 \cdot 60}\right)^{\circ}$

$50^{\circ} + .1^{\circ} + \left(\frac{7}{1200}\right)^{\circ}$

$\approx 50^{\circ} + .1^{\circ} + .005833^{\circ}$

50.105833° or 50.11°

Example: Change 61°42′ 21″ into degrees.

Answer:

Change *Degrees* into *Degrees° Minutes′ Seconds″*

Example: Change 21.256° into Degrees°Minutes′Seconds″
Answer:
$21^\circ + .256^\circ$
$21^\circ + .256(60)'$
$21^\circ + 15.36'$
$21^\circ + 15' + .36'$
$21^\circ + 15' + .36(60)''$
$21^\circ + 15' + 21.6''$
$\approx 21^\circ\, 15'\, 22''$

Example: Change 61.24° into DMS
Answer:

In many applications, like describing the exact location of a star or the exact position of a boat at sea, angles measured in degrees, minutes, and seconds are used. These are usually changed to degrees in decimal form.

1.1 Angles and their Measure Homework

Lial page 7 problems 1-21 odd, 23-28
Sullivan:
In problems 11-21, draw each angle.

11. 30° 13. 135° 15. 450° 17. $\frac{3\pi}{4}$ 19. $-\frac{\pi}{6}$ 21. $\frac{16\pi}{3}$

Convert each angle to a decimal in degrees. Round your answer to two decimal places.

23. 40°10′25″ 25. 1°2′3″ 27. 9°9′9″

Convert each angle to $D°M'S''$ form. Round your answer to the nearest second.

29. 40.32° 31. 18.255° 33. 19.99°

Math 182—Spring 2010
Plane Trigonometry Outline

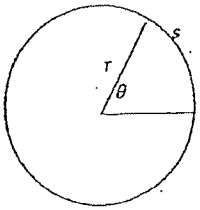
Due Date — *Chapter 3 – Trigonometric Identities* — Page

Chapter 4 – Advanced Trigonometry

MAT 182 Final Exam — Wednesday, May 12
10:00 - 11:50 am in Room B210

1.2 Angles in Degrees and Angles in Radians

In calculus, angles are measured using ***radians.***



A central angle is an angle whose vertex is at the center of a circle.

One radian is the measure of a central angle θ that cuts an arc s which is as long as the radius r of the circle: $s = r$
That angle is one radian, around 57°.

If the angle is 1 radian, then the length of the arc is also 1 radian:
ie when the arc length = radius, then the angle is 1 radian or about 57°.

If an angle θ is in degrees, use the number and the degree symbol when writing a trigonometric function like $\sin 30^\circ$ and $\tan 45^\circ$. If an angle θ is measured in radians, then no symbol is used to represent radians when writing a trigonometric function like $\cos\pi$ and $\sec\frac{\pi}{3}$.

Greek letters: α alpha β beta θ theta φ phi ω omega π = pi ≈ 3.14159265 radians

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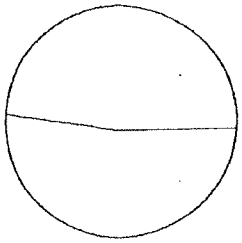
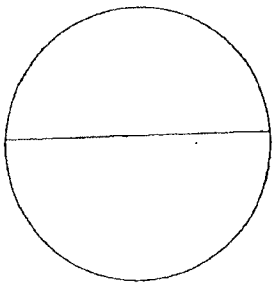
1 revolution $= 2\pi$ radians

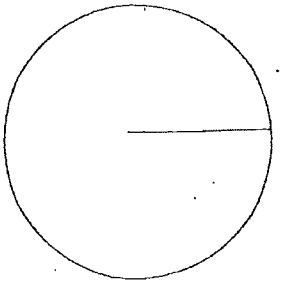
$\frac{1}{2}$ revolution $= \pi$ radians



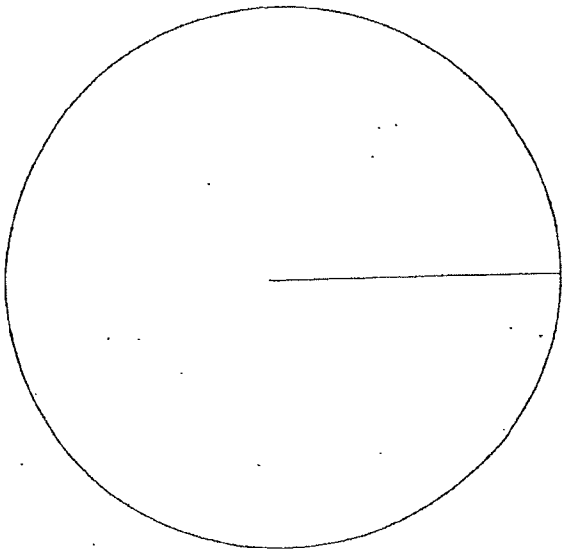
Divide π by 4 and label the angle.
$\frac{1\pi}{4}$ $\frac{2\pi}{4}$ $\frac{3\pi}{4}$ $\frac{4\pi}{4}$



Divide π by 3 then divide π by 6.



Use the circle to mark the degree and radian measure of common angles.
Divide the circle into multiples of 30°, then divide the circle into multiples of 45°.
Transfer this information to the table provided.



Degrees	0°	30°	45°	60°	90°	120°	135°	150°	180°
Radians									

Degrees	210°	225°	240°	270°	300°	315°	330°	360°
Radians								

Change *Degrees* into *radians*:

180 degrees = π radians

1 degree = $\frac{\pi}{180}$ radians

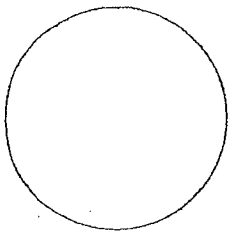
Solve for 1 degree by dividing both sides by 180

Change *Radians* into *degrees*

π radians = 180 degrees

1 radian = $\frac{180}{\pi}$ degrees $\approx 57.3^\circ$

Solve for 1 radian by dividing both sides by π



Example: Convert 60° into radians.

Answer: $60^\circ = 60(1 \text{ degree}) = \frac{60}{1} \cdot \frac{\pi}{180} = \frac{\pi}{3}$ radians

Example: Change 107° into radians.

Example: Convert $\frac{\pi}{6}$ *radians* into degrees.

Answer: $\frac{\pi}{6}$ *radians* $= \frac{\pi}{6}(1 \text{ radian}) = \frac{\pi}{6} \cdot \frac{180}{\pi}$ *degrees* $= 30$ *degrees* ie 30°

Example: Change 3 *radians* into degrees.

1.2 Angles in Degrees and Angles in Radians Homework

Convert each angle in degrees to radians. Express your answer as a multiple of π.

35. 30° 37. 240° 39. -60° 41. 180° 43. -135° 45. -90°

Convert each angle in radians to degrees.

47. $\frac{\pi}{3}$ 49. $-\frac{5\pi}{4}$ 51. $\frac{\pi}{2}$ 53. $\frac{\pi}{12}$ 55. $-\frac{\pi}{2}$ 57. $-\frac{\pi}{6}$

Convert each angle in degrees to radians. Round answer to two decimal places.

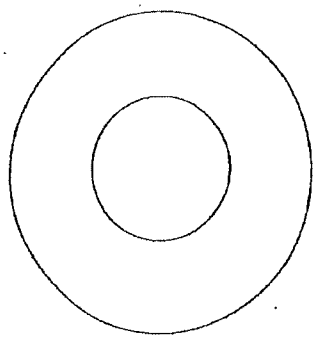
59. 17° 61. -40° 63. 125°

Convert each angle in radians to degrees. Round answer to two decimal places.

65. 3.14 67. 2 69. 6.32

1.3 Arc length, Area, and Velocity

Arc Length: The length of an arc l made by a central angle is found by multiplying the radius times the angle measure. $l = r\theta$



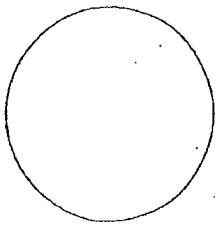
$$\frac{\angle \text{ of } Big\ circle\ \theta}{\angle \text{ of } small\ circle\ "1"} = \frac{Big\ arc\ length\ l}{small\ arc\ length\ "r"}$$

If the small circle's angle is 1 radian (ie 57°), then the length of the arc of the small circle is r, it's own radius.

In other words, $\frac{\theta}{1} = \frac{length\ of\ arc}{r}$

Solving this equation for the length of the arc l gives us the equation for arc length, $l = r\theta$

Example: Find the length of the arc of a circle whose radius is 2 meters and the central angle is .25 radian (ie ¼ radian).



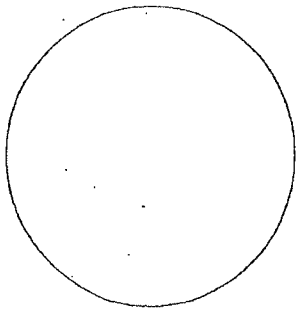
Answer: The arc length $l = r\theta$

$s = (2\text{meters})(.25)$

$s = (2)(¼)$

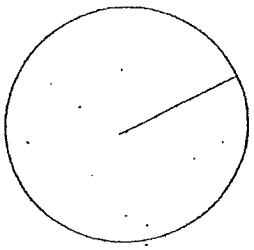
$s = .5$ meter or ½ meter.

Example: Find the length of the arc of a circle whose radius is 10 meters and the central angle is .5 radian (ie ½ radian).



Area of a sector $A=\frac{1}{2}r^2\theta$

A, the area of a slice of pie (ie a sector of a circle formed by a central angle) is found by multiplying $\frac{1}{2}$ times the radius squared times the angle in radians.



$$\frac{\textit{Big angle}}{\textit{small angle}} = \frac{\textit{Big area}}{\textit{small area}}$$

$$\frac{\theta}{\theta} = \frac{A}{A}$$

Suppose the small angle is 2π. The area of the circle is πr^2

$$\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$$

Solving this equation for the area A gives us the equation for area, $A=\frac{1}{2}r^2\theta$

Example: Find the area of the sector with radius 2 feet and an angle of 30°.

Answer: $A = \frac{1}{2}r^2\theta$

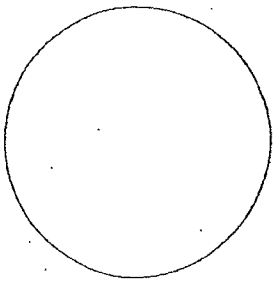
$$A = \frac{1}{2}\left(\frac{2}{1}\right)^2\left(\frac{\pi}{6}\right)$$

$$A = \frac{1}{2}\left(\frac{4}{1}\right)\left(\frac{\pi}{6}\right)$$

$$A = \frac{\pi}{3}\ square\ feet$$

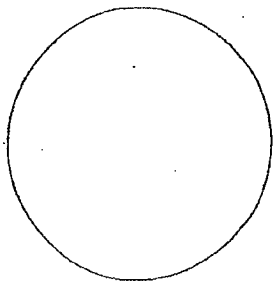
$A \approx 1.05$ square feet

Draw the area of the sector.



Example: Find the area of the sector with radius 6 feet and an angle of 2 radians.

Answer: Write the formula and draw the area of the sector.



Distance = (rate) (time) $d = r \cdot t \Rightarrow rate = \frac{distance}{time}$ ie $r = \frac{d}{t}$

Linear speed $= v$ of an object traveling in a circle is found by dividing the arc length s by the time t: $v = \frac{s}{t}$ length per unit of time, like feet per second or miles per hour

$$linear\ speed = v = \frac{arc\ length}{unit\ of\ time}$$

Angular speed $= \omega = \frac{angle\ in\ radians}{unit\ of\ time}$

Angular speed is how the turning rate of an engine is described. For instance, an engine idling at 900 rpm (revolutions per minute) is one that rotates at an angular speed of $900 \frac{revolutions}{minute} = \frac{900\ revolutions}{minute} \cdot \frac{2\pi\ radians}{1\ revolution} = 1800\pi \frac{radians}{minute}$

Since angular speed, ω, is given in terms of *revolutions* per unit of time, you must convert it to *radians* per unit of time.

There is an important relationship between linear speed and angular speed:

$linear\ speed = v = \frac{s}{t} = r \cdot \omega$ so $v = r \cdot \omega$

Remember that linear speed $v = \frac{s}{t}$ has the dimensions of length per unit of time (such as feet per second or miles per hour

In $v = r \cdot \omega$, the radius of the circular motion r has the same length dimension as s, *ie if s is in feet then r is also in feet.*

Summary of formulas

distance = (rate) (time) $d = r \cdot t \Rightarrow rate = \frac{distance}{time}$ ie $r = \frac{d}{t}$

180 degrees = π radians

Arc length $s = r \cdot \theta$ = (the length of the radius)(the size of the angle in radians)

Area of a sector $A = \frac{1}{2} r^2 \theta = (\frac{1}{2})$ (the radius squared)(the angle in radians)

Linear speed $v = \frac{s}{t} = \frac{length\ of\ the\ arc}{unit\ of\ time}$ like feet per second or miles per hour

Also, $v = r\ \omega$ = (radius of the circular motion)(angular speed)

Angular speed $\omega = \frac{\theta}{t} = \frac{central\ angle\ in\ radians}{time\ elapsed}$ = radians per unit of time

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1.3 Arc length, Area, and Velocity Homework

L denotes the length of the arc of a circle of radius r subtended by the central angle θ. Find the missing quantity rounded to three decimal places.

71. $r = 10$ meters, $\theta = \frac{1}{2}$ radian, $L = ?$

73. $\theta = \frac{1}{3}$ radian, $L = 2$ feet, $r = ?$

75. $r = 5$ miles, $L = 3$ miles, $\theta = ?$

77. $r = 2$ inches, $\theta = 30^\circ$, $L = ?$

In problems 79-85, A denotes the area of the sector of a circle of a radius r formed by the central angle θ. Find the missing quantity rounded to three decimal places.

79. $r = 10$ meters, $\theta = \frac{1}{2}$ radian, $A = ?$

81. $\theta = \frac{1}{3}$ radian, $A = 2$ square feet, $r = ?$

83. $r = 5$ miles, $A = 3$ square miles, $\theta = ?$

85. $r = 2$ inches, $\theta = 30^\circ$, $A = ?$

87. Draw a circle with angle $\frac{\pi}{3}$ and radius 2 ft. Find the arc length L and the area A, *both* rounded to three decimal places.

88. Draw a circle with angle $\frac{\pi}{6}$ and radius 4 m. Find the arc length L and the area A, *both* rounded to three decimal places.

89. Draw a circle with angle 70° and radius 12 yds. Find the arc length L and the area A, *both* rounded to three decimal places.

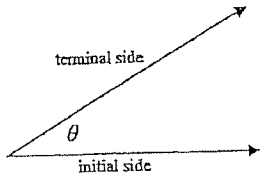
90. Draw a circle with angle 50° and radius 9 cm. Find the arc length L and the area A, *both* rounded to three decimal places.

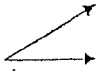
1.4 Right-Triangle Trigonometry

A right triangle has the famous Pythagorean Theorem related to it: $a^2+b^2=c^2$

$(leg)^2+(leg)^2=(hypotenuse)^2$

Draw the rectangular axes so the *acute* angles shown below have the vertex of each angle at the origin. Form two right-triangles by dropping a perpendicular line from the tip of the terminal side to the x-axis. The triangles are similar triangles so they have proportionate sides. Label the sides of the larger triangle 9, 12, and 15, and the sides of the smaller triangle 3, 4, and 5.





The trigonometry functions are ratios of *the lengths* of two sides of a right triangle. Each ratio depends on the size of the angle θ and not the length of the sides of the triangle. Note that in each triangle, the ratio formed by the terminal side over the initial side is $\frac{5}{4}$

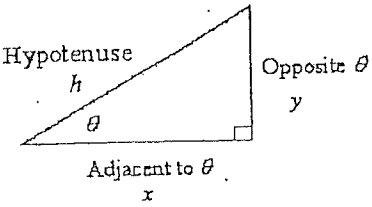
The six trigonometric functions are: $\sin\theta$ $\cos\theta$ $\tan\theta$ $\csc\theta$ $\sec\theta$ $\cot\theta$

$\sin\theta$ is read "the sine of theta."

You cannot separate them. The $\sin\theta$ is a ratio. However, sin without θ is a "pecado," a sin, not a ratio. Therefore, always *write the trig function with the angle.*

$\sin\theta$

The angle is called the argument of the function.



The ratio called $\sin\theta$ is formed by the *length* of the side opposite the angle over the *length* of the side called the hypotenuse: $\sin\theta=\frac{Opposite}{hypotenuse}$

The ratio called $\cos\theta$ is formed by the *length* of the side adjacent (next to) the angle over the *length* of the side called the hypotenuse: $\cos\theta=\frac{Adjacent}{hypotenuse}$

The ratio called $\tan\theta$ is formed by the *length* of the side opposite the angle over the *length* of the side adjacent (next to) the angle: $\tan\theta=\frac{Opposite}{Adjacent}$

When graphing, the $\sin\theta$ is referred to as the "*y-coordinate*" and the $\cos\theta$ is referred to as the "*x-coordinate.*" Therefore, a point has the coordinates $(\cos\theta, \sin\theta)$ instead of (x, y).

"O" and "y" are the side opposite the angle. "A" and "x" are the side adjacent (next) to the angle.

The hypotenuse "h" is also known as the radius "r" of the right triangle.

Each ratio has a name that depends on the angle θ. The 6 ratios are formed as described below.

sine of $\theta = \dfrac{Opposite}{hypotenuse}$	cosine of $\theta = \dfrac{Adjacent}{hypotenuse}$	tangent of $\theta = \dfrac{Opposite}{Adjacent}$
$\sin\theta = \dfrac{O}{h} = \dfrac{y}{r}$	$\cos\theta = \dfrac{A}{h} = \dfrac{x}{r}$	$\tan\theta = \dfrac{O}{A} = \dfrac{y}{x}$
cosecant of $\theta = \dfrac{hypotenuse}{Opposite}$	secant of $\theta = \dfrac{hypotenuse}{Adjacent}$	cotangent of $\theta = \dfrac{Adjacent}{Opposite}$
$\csc\theta = \dfrac{h}{O} = \dfrac{r}{y}$	$\sec\theta = \dfrac{h}{A} = \dfrac{r}{x}$	$\cot\theta = \dfrac{A}{O} = \dfrac{x}{y}$

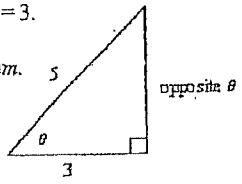
Notice that the reciprocal of the sine function is the cosecant function. The reciprocal identities are provided below.

Reciprocal Identities $\csc\theta = \dfrac{1}{\sin\theta}$ $\sin\theta = \dfrac{1}{\csc\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cos\theta = \dfrac{1}{\sec\theta}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$ $\tan\theta = \dfrac{1}{\cot\theta} = \dfrac{\sin\theta}{\cos\theta}$ Memorize these reciprocal identities.

Example 1 Find the value of the six trigonometric functions of the angle θ in the triangle.

We see that the hypotenuse $= 5$ and the adjacent side is $= 3$.



To find the length of the opposite side, we use the Pythagorean Theorem.

$a^2 + b^2 = c^2$

$(leg)^2 + (leg)^2 = (hypotenuse)^2$

$(\text{adjacent})^2 + (\text{opposite})^2 = (\text{hypotenuse})^2$

$3^2 + (\text{opposite})^2 = 5^2$

$(\text{opposite})^2 = 25 - 9 = 16$

$\text{opposite} = 4$

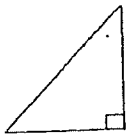
Now that we know the lengths of the three sides, we can find the six trigonometric functions:

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{4}{5}$ $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{3}{5}$ $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{4}{3}$

$\csc\theta = \dfrac{\text{hypotenuse}}{\text{opposite}} = \dfrac{5}{4}$ $\sec\theta = \dfrac{\text{hypotenuse}}{\text{adjacent}} = \dfrac{5}{3}$ $\cot\theta = \dfrac{\text{adjacent}}{\text{opposite}} = \dfrac{3}{4}$

EXAMPLE 2 $\sin\theta = \frac{\sqrt{5}}{5}$ Find the remaining 5 trigonometric functions of θ.

Solution: $\cos\theta =$ **Label the sides of a right triangle:**



$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{5}}{5}}{\frac{2\sqrt{5}}{5}} = \frac{1}{2}$$

$a^2 + b^2 = c^2$

$\csc\theta = \frac{1}{\sin\theta} =$ $\sec\theta = \frac{1}{\cos\theta} =$

$\cot\theta = \frac{1}{\tan\theta} =$

Steps for Finding the Values of 5 Trigonometric Functions When One is Known

Step 1: Draw a right triangle showing the angle θ and label two sides of the triangle.
Step 2: Find the length of the third side by using the Pythagorean Theorem.
Step 3: Use the definitions to find the value of the remaining 5 trigonometric functions.

Example 4 $\sin\theta = \frac{1}{3}$. Find the exact value of the remaining five trigonometric functions.

Solution: *1)* Draw and label the sides of a right triangle in Quadrant 1.
2) Use the Pythagorean Theorem to find the adjacent side.

$a^2 + b^2 = c^2$

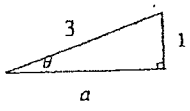
$(adjacent\ side)^2 + (opposite\ side)^2 = (hypotenuse)^2$

$(\text{adjacent})^2 + 1^2 = 3^2$

$(\text{adjacent})^2 + 1 = 9$

$(\text{adjacent})^2 = 8$

$\text{adjacent} = 2\sqrt{2}$



3) Use the definitions to find the value of the remaining five trigonometric functions.

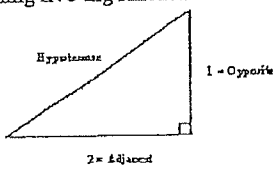
$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} =$ $\tan\theta = \frac{\text{opposite}}{\text{adjacent}} =$

$\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} =$ $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} =$ $\cot\theta = \frac{\text{adjacent}}{\text{opposite}} =$

$\frac{2\sqrt{2}}{3}$ $\frac{\sqrt{2}}{4}$ 3 $\frac{3\sqrt{2}}{4}$ $2\sqrt{2}$

Example 5 If $\tan\theta = \frac{1}{2}$, find the exact value of the remaining five trig functions.

1) Draw the right triangle where $\tan\theta = \frac{1}{2} = \frac{\text{opposite}}{\text{adjacent}}$



2) Use the Pythagorean Theorem to find the hypotenuse.

3) Now use the definitions to find the value of the other 5 trigonometric functions.

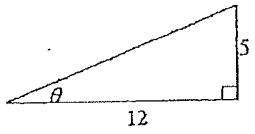
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} =$ $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} =$

$\csc\theta = \frac{\text{hypotenuse}}{\text{adjacent}} =$ $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} =$ $\cot\theta = \frac{\text{adjacent}}{\text{opposite}} =$

1.4 Right-Triangle Trigonometry Homework

Find the value of the six trigonometric functions in triangles 11-19.

11.



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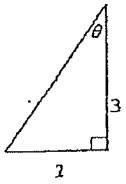

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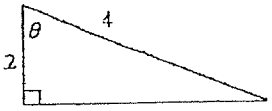
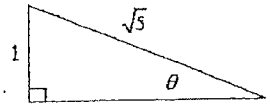
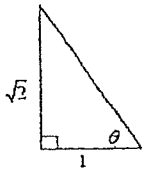


17.



19.



For #21 and 23, use identities to find the value of the remaining trigonometric functions.

21. $\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$

23. $\sin\theta = \frac{2}{3}$, $\cos\theta = \frac{\sqrt{5}}{3}$

In problems 25-35, find the value of the remaining five trigonometric functions.

25. $\sin\theta = \frac{\sqrt{2}}{2}$ 27. $\cos\theta = \frac{1}{3}$ 29. $\tan\theta = \frac{1}{2}$ 31. $\sec\theta = 3$ 33. $\tan\theta = \sqrt{2}$ 35. $\csc\theta = 2$

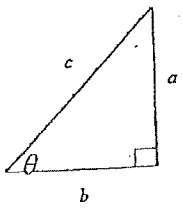
1.5 The Pythagorean Identity and Complementary Functions

Pythagorean Identities

You now know enough so we can derive the Pythagorean Identities.

The Pythagorean Theorem states that $a^2 + b^2 = c^2$, which we can write as

$$(\text{opposite})^2 + (\text{adjacent})^2 = (\text{hypotenuse})^2$$



Dividing each side by the $(\text{hypotenuse})^2$ we get

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2} \quad \text{or} \quad \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$$

$$(\sin\theta)^2 + (\cos\theta)^2 = 1$$

The Pythagorean Identity $\sin^2\theta + \cos^2\theta = 1$

Divide the Pythagorean Identity by $\cos^2\theta$ and simplify.

$$\sin^2\theta + \cos^2\theta = 1$$

Divide the Pythagorean Identity by $\sin^2\theta$ and simplify.

$$\sin^2\theta + \cos^2\theta = 1$$

PYTHAGOREAN IDENTITIES Commit these Pythagorean Identities to memory.

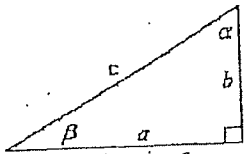
$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $\cot^2\theta + 1 = \csc^2\theta$

Recall that two acute angles are called **complementary** if they add up to 90 degrees. Also, the three angles of any triangle add up to 180°. So in a right triangle, the two acute angles are complementary angles. Additionally:

The sine and cosine are called **cofunctions.**

The cosecant and secant are cofunctions.

The tangent and cotangent are cofunctions.



Which leads us to ↓

The Complementary Angle Theorem: **Cofunctions of complementary angles** are ***equal.***

$$\sin\beta = \frac{opposite}{hypotenuse} = \cos\alpha$$

$$\cos\beta = \frac{adjacent}{hypotenuse} = \sin\alpha$$

Complementary angles

$\sin 30^\circ = \cos 60^\circ$

Cofunctions

$$\tan\beta = \frac{opposite}{adjacent} = \cot\alpha$$

$$\csc\beta = \frac{hypotenuse}{opposite} = \sec\alpha$$

Complementary angles

$\tan 40^\circ = \cot 50^\circ$

Cofunctions

$$\sec\beta = \frac{hypotenuse}{adjacent} = \csc\alpha$$

$$\cot\beta = \frac{adjacent}{opposite} = \tan\alpha$$

Complementary angles

$\sec 80^\circ = \csc 10^\circ$

Cofunctions

Because of these relationships, the sine and cosine, the tangent and cotangent, and the secant and cosecant are called **cofunctions.**

Example 6 Using the Complementary Angle Theorem, find the corresponding cofunctions.

$\sin 62^\circ = \cos(90^\circ - 62^\circ) = \cos$ ____

$\cos\frac{\pi}{4} = \sin$ ____

$\tan\frac{\pi}{2} = \cot$ ____

$\csc\frac{\pi}{6} = \sec$ ____

Example 7 Find the value of each expression. Do not use a calculator.

(a) $\sec 28^\circ - \csc 62^\circ$ (b) $\dfrac{\sin 35^\circ}{\cos 55^\circ}$

1.5 The Pythagorean Identity and Complementary Functions Homework

In Problems 37-53, find the exact value of each expression. *Do not use a calculator.*

37. $\sin^2 20^\circ + \cos^2 20^\circ$ 39. $\sin 80^\circ - \cos 52^\circ$ 41. $\tan 50^\circ - \dfrac{\sin 50^\circ}{\cos 50^\circ}$ 43. $\sin 38^\circ - \cos 52^\circ$

45. $\dfrac{\cos 10^\circ}{\sin 80^\circ}$ 47. $1 - \cos^2 20^\circ - \cos^2 70^\circ$ 49. $\tan 20^\circ - \dfrac{\cos 70^\circ}{\cos 20^\circ}$

51. $\tan 35^\circ \cdot \sec 55^\circ \cdot \cos 35^\circ$ 53. $\cos 35^\circ \sin 55^\circ + \cos 55^\circ \sin 35^\circ$

55. Given $\sin 30^\circ = \dfrac{1}{2}$, use trigonometric identities to find the exact value of

(a) $\cos 60^\circ$ (b) $\cos^2 30^\circ$ (c) $\csc \dfrac{\pi}{6}$ (d) $\sec \dfrac{\pi}{3}$

57. Given $\tan\theta = 4$, use trigonometric identities to find the exact value of

(a) $\sec^2\theta$ (b) $\cot\theta$ (c) $\cot\left(\dfrac{\pi}{2} - \theta\right)$ (d) $\csc^2\theta$

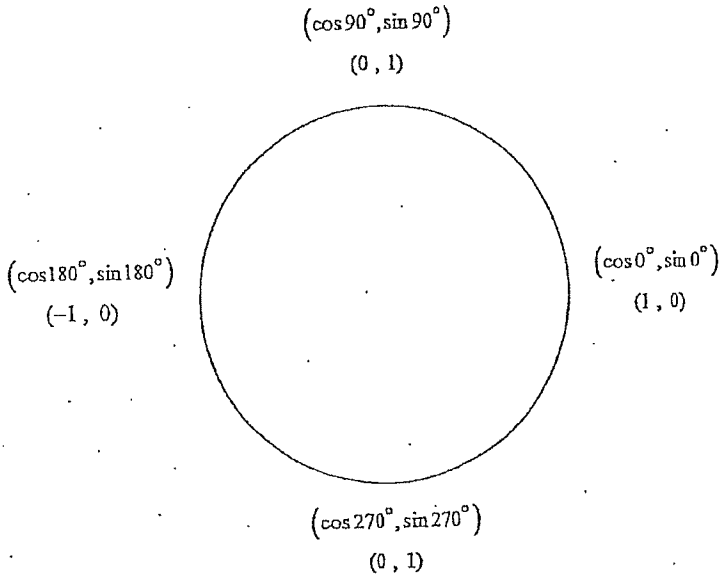
59. Given $\csc\theta = 4$, use trigonometric identities to find the exact value of

(a) $\sin\theta$ (b) $\cot^2\theta$ (c) $\sec(90^\circ - \theta)$ (d) $\sec^2\theta$

1.6 Computing the Values of Trig Functions for Special Triangles

The special angles in trigonometry are $0°,\ 30°,\ 45°,\ 60°,\ 90°,\ 180°,\ 270°,\ 360°$

In radian measure these angles are $0,\ \frac{\pi}{6},\ \frac{\pi}{4},\ \frac{\pi}{3},\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{4},\ 2\pi$

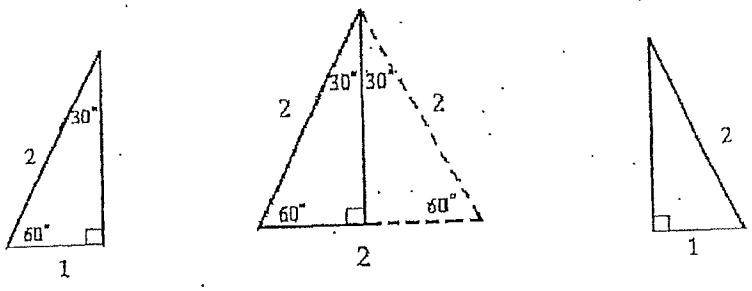


Commit to memory the values of the 6 trigonometric functions for angles $0°,\ 90°,\ 180°,\ 270°, 360°$

$\sin 0° = 0$	$\sin 90° = 1$	$\sin 180° = 0$	$\sin 270° = -1$	$\sin 360° = 0$
$\cos 0° = 1$	$\cos 90° = 0$	$\cos 180° = -1$	$\cos 270° = 0$	$\cos 360° = 1$
$\csc 0° = \text{undef}$	$\csc 90° = 1$	$\csc 180° = \text{undef}$	$\csc 270° = -1$	$\csc 360° = \text{undef}$
$\sec 0° = 1$	$\sec 90° = \text{undef}$	$\sec 180° = -1$	$\sec 270° = \text{undef}$	$\sec 360° = 1$
$\tan 0° = 0$	$\tan 90° = \text{undef}$	$\tan 180° = 0$	$\tan 270° = \text{undef}$	$\tan 360° = 0$
$\cot 0° = \text{undef}$	$\cot 90° = 0$	$\cot 180° = \text{undef}$	$\cot 270° = 0$	$\cot 360° = \text{undef}$

Consider an equilateral triangle cut in half so the top 60° angle makes two 30°– angles.

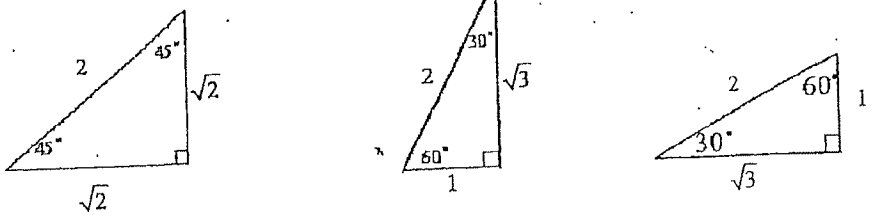
Each right triangle is now a special triangle known as a 30° – 60° – 90° triangle.



The sides of the right triangles measure 1, 2, $\sqrt{3}$. The hypotenuse measures 2.
The side opposite the 30° measures 1, and the side opposite the 60° angle measures $\sqrt{3}$.

Commit to memory the trigonometric function values of the two very special triangles often used in trigonometry. They are the

$45°\ 45°\ 90°$ or $\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}$ and the $30°\ 60°\ 90°$ or $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$



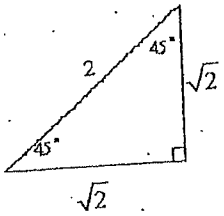
$\sin 45° = \frac{\sqrt{2}}{2}$ $\csc 45° = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$ $\sin 30° = \frac{1}{2}$ $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 45° = \frac{\sqrt{2}}{2}$ $\sec 45° = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$ $\cos 30° = \frac{\sqrt{3}}{2}$ $\cos 60° = \frac{1}{2}$

$\tan 45° = \frac{\sqrt{2}}{\sqrt{2}} = 1$ $\cot 45° = \frac{\sqrt{2}}{\sqrt{2}} = 1$ $\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\tan 60° = \sqrt{3}$

Example 1 Find the values of the six trigonometric functions of $\frac{\pi}{4} = 45^\circ$.

Solution Draw the triangle and label the sides and the angles.



$\sin\frac{\pi}{4} = \sin 45^\circ =$

$\cos\frac{\pi}{4} =$

$\sec\frac{\pi}{4} =$

$\csc\frac{\pi}{4} =$

$\tan\frac{\pi}{4} =$

$\cot\frac{\pi}{4} =$

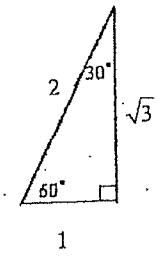
Example 2 Find the exact value of each expression.

a) $(\sin 45^\circ)(\tan 45^\circ)$

b) $\left(\sec\frac{\pi}{4}\right)\left(\cot\frac{\pi}{4}\right)$

Example 3 Find the Exact Values of the six trigonometric functions of $\frac{\pi}{6}=30^\circ$ and $\frac{\pi}{3}=60^\circ$

Solution: Form a right triangle and label the sides and the angles appropriately.



$\sin\frac{\pi}{6}=$ $\cos\frac{\pi}{3}=$

$\cos\frac{\pi}{6}=$ $\sin\frac{\pi}{3}=$

$\tan\frac{\pi}{6}=$ $\cot\frac{\pi}{3}=$

$\csc\frac{\pi}{6}=$ $\sec\frac{\pi}{3}=$

$\sec\frac{\pi}{6}=$ $\csc\frac{\pi}{3}=$

$\cot\frac{\pi}{6}=$ $\tan\frac{\pi}{3}=$

Example 4 Find the exact value of each expression.

(*a*) $\sin 45^\circ \cos 30^\circ$ (*b*) $\tan\frac{\pi}{4}-\sin\frac{\pi}{3}$ (*c*) $\tan^2\frac{\pi}{6}+\sin^2\frac{\pi}{4}$

Example 5 Use a calculator to find the approximate value (to two decimal places) of:

(*a*) $\cos 48^\circ$ (*b*) $\csc 21^\circ$ (*c*) $\tan\frac{\pi}{12}$

Solution Since these are not the special angles, we will use the calculator as follows.

(a) First, we set the MODE on the calculator to receive degrees.

(b) Most calculators do not have a csc key. However, the $\csc 21^\circ$ is $\frac{1}{\cos 21^\circ}$.

(c) Set the MODE on the calculator to receive radians.

1.6 Computing the Values of Trig Functions for Special Triangles Homework

In problems 7-16, $f(\theta)=\sin\theta$ and $g(\theta)=\cos\theta$. Find the value of each expression if $\theta=60^\circ$. Do not use a calculator.

7. $f(\theta)$ 9. $f\left(\frac{\theta}{2}\right)$ 11. $\left[f(\theta)\right]^2$ 13. $2f(\theta)$ 15. $\frac{f(\theta)}{2}$

In problems 17-27, find the exact value of each expression. *Do not use calculator.*

17. $4\cos 45^\circ - 2\sin 45^\circ$ 19. $6\tan 45^\circ - 8\cos 60^\circ$ 21. $\sec\frac{\pi}{4}+2\csc\frac{\pi}{3}$

23. $\sec^2\frac{\pi}{6}-4$ 25. $\sin^2 30^\circ+\cos^2 60^\circ$ 27. $1-\cos^2 30^\circ-\cos^2 60^\circ$

In problems 29-45, use a calculator to find the approximate value of each expression. Round the answer to two decimal places.

29. $\sin 28^\circ$ 31. $\tan 21^\circ$ 33. $\sec 41^\circ$ 35. $\sin\frac{\pi}{10}$ 37. $\tan\frac{5\pi}{12}$

39. $\sec\frac{\pi}{12}$ 41. $\sin 1$ 43. $\sin 1^\circ$ 45. $\tan 0.3$

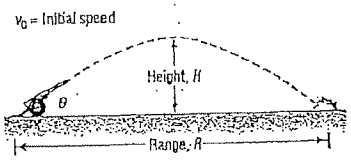
47. Applications and Extensions Find the range R and maximum height H rounded to 2 decimal places.

Projectile Motion The path of a projectile fired at an inclination θ to the horizontal with initial speed v_0 is a parabola (see the figure). The range R of the projectile, that is, the horizontal distance that the projectile travels, is found using the function

$$R(\theta)=\frac{2v_0^2\sin\theta\cos\theta}{g}$$

Where $g\approx 32.2$ feet per second ≈ 9.8 meters per second is the acceleration due to gravity. The maximum height H of the projectile is given by the function

$$H(\theta)=\frac{v_0^2\sin^2\theta}{2g}$$



Find the range R and maximum height H to two decimal places when the projectile is fired at an angle of 45° to the horizontal with an initial speed of 100 feet per second.

1.7 Trigonometric Functions of General Angles

Example 1 Find the value of the six trigonometric functions of a positive angle θ if (4, -3) is a point on its terminal side. Hint: Start by drawing the angle and the corresponding triangle. Label the sides.

Example 2 Find the values of the six trigonometric functions of

(a) $\theta = 0 = 0^\circ$ (b) $\theta = \frac{\pi}{2} = 90^\circ$ (c) $\theta = \pi = 180^\circ$ (d) $\theta = \frac{3\pi}{2} = 270^\circ$

Answers:

a) $\sin 0 = \sin 0^\circ = 0$, $\cos 0 = 1$, $\tan 0 = 0$, $\sec 0 = 1$, $\csc 0$ & $\cot 0$ are not defined

b) $\sin \frac{\pi}{2} = 1$, $\cos \frac{\pi}{2} = 0$, $\csc \frac{\pi}{2} = 1$, $\cot \frac{\pi}{2} = 0$, $\tan \frac{\pi}{2}$ and $\sec \frac{\pi}{2}$ are not defined

c) $\sin \pi = 0$, $\cos \pi = -1$, $\tan \pi = 0$, $\sec \pi = -1$, $\csc \pi$ & $\cot \pi$ are not defined

d) $\sin \frac{3\pi}{2} = -1$, $\cos \frac{3\pi}{2} = 0$, $\csc \frac{3\pi}{2} = -1$, $\cot \frac{3\pi}{2} = 0$, $\tan \frac{3\pi}{2}$ & $\sec \frac{3\pi}{2}$ are not defined

Example 3 a) $\sin 390°$ b) $\cos 420°$ c) $\tan\frac{9\pi}{4}$ d) $\sec\left(-\frac{7\pi}{4}\right)$ e) $\csc(-270°)$

Hint: It is best to sketch the angle first.

a) The angle is coterminal with $30°$.

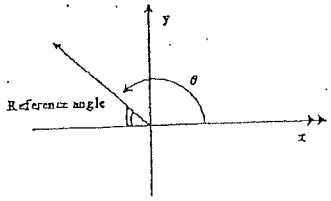
$\therefore \sin 390° = \sin(360° + 30°) = \sin 30° = \frac{1}{2}$

b) c) d) e)

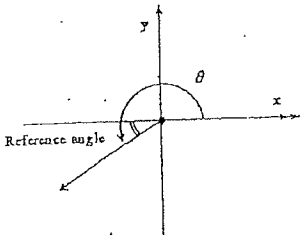
Example 4 Name the Quadrant in Which the Angle Lies

(a) $\sin\theta < 0$ and $\cos\theta < 0$ (b) $\sin\theta > 0$ and $\cos\theta < 0$

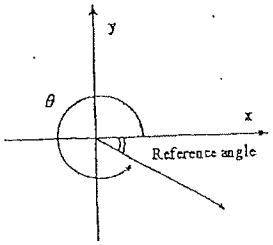
(c)



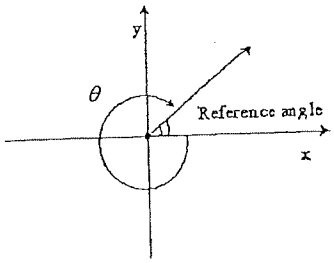
(d)



(e)



(f)

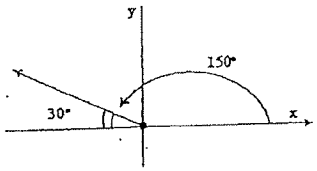


Example 5: Find the sine of the reference angle for each of the following angles:

(a) 150° (b) -45° (c) $\frac{9\pi}{4}$ (d) $-\frac{5\pi}{6}$

Solution (a) $\sin 30^\circ = \frac{1}{2}$

(b)



(c)

(d)

Steps for finding the values of the trigonometric functions of an Angle

Step 1: If the angle θ is on the x-axis or y-axis, label a point on its terminal side, and name the trig values.

Step 2: If the angle θ is in a quadrant, draw a triangle and label its sides, then name the trig values.

Example 6: Find the value of the following trigonometric functions.

(a) $\sin 135^\circ$ (b) $\cos 600^\circ$ (c) $\cos \frac{17\pi}{6}$ (d) $\tan\left(\frac{\pi}{-3}\right)$

Example 7: Given that $\cos\theta = -\frac{2}{3}, \frac{\pi}{2} < \theta < \pi$, find the value of the remaining trig functions.

Example 8: If $\tan\theta = -4$ and $\sin\theta < 0$, find the value of the remaining trigonometric functions.

1.7 Trigonometric Functions of General Angles Homework

In problems 11-20, a point on the terminal side of an angle θ is given. Find the value of the six trigonometric functions.

11. $(-3,4)$ 13. $(2,-3)$ 15. $(-3,-3)$ 17. $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$ 19. $\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}$

For #21-32, find the value of each expression. Do not use a calculator.

21. $\sin 405^\circ$ 23. $\tan 405^\circ$ 25. $\csc 450^\circ$ 27. $\cot 390^\circ$ 29. $\cos\frac{33\pi}{4}$ 31. $\tan(21\pi)$

In Problems 33-39, name the quadrant in which the angle θ lies.

33. $\sin\theta > 0, \cos\theta < 0$ 35. $\sin\theta < 0, \tan\theta < 0$ 37. $\cos\theta > 0, \cot\theta < 0$ 39. $\sec\theta < 0, \tan\theta > 0$

In Problems 41-57, name the reference angle of each angle. 41. -30° 43. 120°

45. 210° 47. $\frac{5\pi}{4}$ 49. $\frac{8\pi}{3}$ 51. 135° 53. $-\frac{2\pi}{3}$ 55. 440° 57. $\frac{15\pi}{4}$

For #59-88, find the value. Do not use the calculator. 59. $\sin 150^\circ$ 61. $\cos 315^\circ$ 63. $\sin 510^\circ$

65. $\cos(-45^\circ)$ 67. $\sec 240^\circ$ 69. $\cot 330^\circ$ 71. $\cos\frac{3\pi}{4}$ 73. $\cot\frac{7\pi}{6}$ 75. $\cos\frac{13\pi}{4}$

77. $\sin\left(-\frac{2\pi}{3}\right)$ 79. $\tan\frac{14\pi}{3}$ 81. $\csc(-315^\circ)$ 83. $\sin(8\pi)$ 85. $\tan(7\pi)$ 87. $\sec(-3\pi)$

In Problems 89-106, find the value of the remaining trigonometric functions.

89. $\sin\theta = \frac{12}{13}$, θ in Quadrant II 91. $\cos\theta = -\frac{4}{5}$, θ in Quadrant III

93. $\sin\theta = \frac{5}{13}$, $90^\circ < \theta < 180^\circ$ 95. $\cos\theta = -\frac{1}{3}, 180^\circ < \theta < 270^\circ$

97. $\sin\theta = \frac{2}{3}$, $\tan\theta < 0$ 99. $\sec\theta = 2$, $\sin\theta < 0$ 101. $\tan\theta = \frac{3}{4}$, $\sin\theta < 0$

103. $\tan\theta = -\frac{1}{3}$, $\sin\theta > 0$ 105. $\csc\theta = -2$, $\tan\theta > 0$

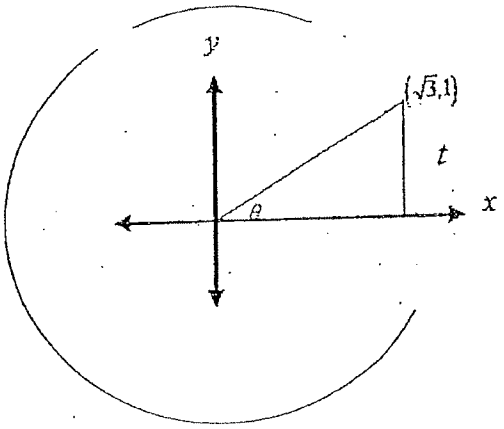
107. Find the exact value of $\sin 45^\circ + \sin 135^\circ + \sin 225^\circ + \sin 315^\circ$

109. If $f(\theta) = \sin\theta = 0.2$, find $f(\theta+\pi)$ 111. If $F(\theta) = \tan\theta = 3$, find $F(\theta+\pi)$.

113. If $\sin\theta = \frac{1}{5}$, find $\csc\theta$.

1.8 Circle Trigonometry

Let t be the length of the arc from the x-axis to the point $P=\left(\sqrt{3}\ ,\ 1\right)$ on a circle.



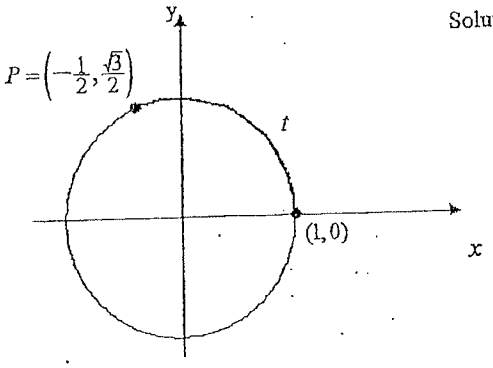
$\theta = t$ *radians*

$\sin\theta = \sin t = \frac{1}{2}$

$\cos\theta = \cos t = \frac{\sqrt{3}}{2}$

Example 1 $P=\left(-1,\sqrt{3}\right)$ Find the value of the corresponding 6 trigonometric functions.

Solution: $\sin t = \frac{\sqrt{3}}{2}$



The unit circle has a radius of 1.

Domain & Range of Trigonometric Functions

Function	Domain is the angle	Range is the function
$f(\theta)=\sin\theta$	θ = All real numbers	$\sin\theta$ = [−1 to 1]
$f(\theta)=\cos\theta$	θ = All real numbers	$\cos\theta$ = [−1 to 1]
$f(\theta)=\tan\theta$	$\theta = (-\infty,+\infty)$, except odd integer multiples of $\frac{\pi}{2}$	$\tan\theta = (-\infty,+\infty)$
$f(\theta)=\csc\theta$	$\theta = (-\infty,+\infty)$, except integer multiples of π	$\csc\theta \le -1,\ \csc\theta \ge 1$
$f(\theta)=\sec\theta$	$\theta = (-\infty,+\infty)$, except odd integer multiples of $\frac{\pi}{2}$	$\sec\theta \le -1,\ \sec\theta \ge 1$
$f(\theta)=\cot\theta$	$\theta = (-\infty,+\infty)$, except integer multiples of π	$\cot\theta = (-\infty,+\infty)$

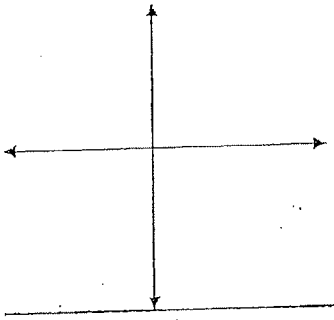
Periodic functions repeat after a certain time.
The trigonometric functions repeat so they are periodic functions.

2π is the repeating period for sine, cosine, cosecant, and secant. *For tangent it is π.*

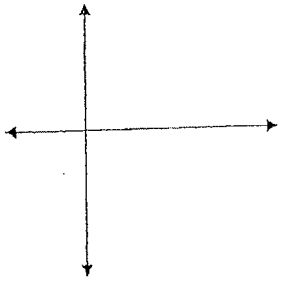
$\sin(\theta+2\pi)=\sin\theta$ $\cos(\theta+2\pi)=\cos\theta$ $\tan(\theta+\pi)=\tan\theta$

$\csc(\theta+2\pi)=\csc\theta$ $\sec(\theta+2\pi)=\sec\theta$ $\cot(\theta+\pi)=\cot\theta$

Example 2 Find the exact value of: a) $\sin 420^\circ$ b) $\tan\frac{5\pi}{4}$ c) $\cos\frac{11\pi}{4}$



Even functions are symmetric about the y-axis, odd functions about the origin.
Recall that a function is even if $f(-x)=f(x)$ The sign of the x changes but the y doesn't.
A function is odd if $-f(-x)=f(x)$ The sign of x and y changes, but the graph doesn't.



$\cos\left(\frac{\pi}{6}\right)$ $\sin\left(\frac{\pi}{6}\right)$

The cosine and the secant are even functions: $\cos(-\theta)=\cos\theta$ $\sec(-\theta)=\sec\theta$

The rest are odd functions: $\sin(-\theta)=-\sin\theta$ $\tan(-\theta)=-\tan\theta$ $\csc(-\theta)=-\csc\theta$ $\cot(-\theta)=-\cot\theta$

Example 3 Find the exact value of: a) $\sin(-45^\circ)$ b) $\cos(-\pi)$ c) $\cot\left(-\frac{3\pi}{2}\right)$ d) $\tan\left(-\frac{37\pi}{4}\right)$

1.8 Circle Trigonometry Homework

For #21-36 find the exact value. Do not use a calculator. 21. $\sin 405^\circ$ 23. $\tan 405^\circ$ 25. $\cos 450^\circ$ 27. $\cot 390^\circ$ 29. $\cos\frac{33\pi}{4}$ 31. $\tan(21\pi)$ 33. $\sec\frac{17\pi}{4}$ 35. $\tan\frac{19\pi}{6}$

For #37-54, find the exact value of each expression. Do not use a calculator.

37. $\sin(-60^\circ)$ 39. $\tan(-30^\circ)$ 41. $\sec(-60^\circ)$ 43. $\sin(-90^\circ)$ 45. $\tan\left(-\frac{\pi}{4}\right)$ 47. $\cos\left(-\frac{\pi}{4}\right)$ 49. $\tan(-\pi)$ 51. $\csc\left(-\frac{\pi}{4}\right)$ 53. $\sec\left(-\frac{\pi}{6}\right)$

In Problems 55-60, find the exact value of each expression. Do not use a calculator.

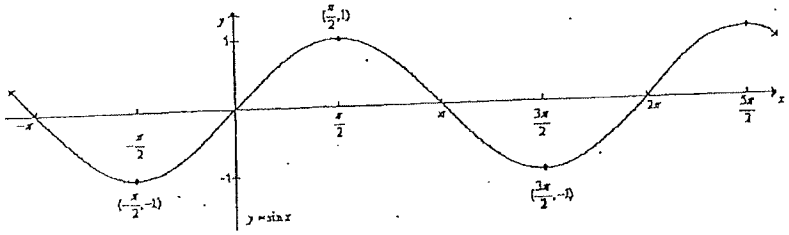
55. $\sin(-\pi)+\cos(5\pi)$ 57. $\sec(-\pi)+\csc\left(-\frac{\pi}{2}\right)$ 59. $\sin\left(-\frac{9\pi}{4}\right)-\tan\left(-\frac{9\pi}{4}\right)$

61. What is the domain of the sine function?
63. For what numbers θ is $f(\theta)=\tan\theta$ not defined?
65. For what numbers θ is $f(\theta)=\sec\theta$ not defined?
67. What is the range of the sine function?
69. What is the range of the tangent function?
71. What is the range of the secant function?
73. Is the sine function even, odd, or neither? Is its graph symmetric? With respect to what?
75. Is the tangent function even, odd, or neither? Is its graph symmetric? With respect to what?
77. Is the secant function even, odd, or neither? Is its graph symmetric? With respect to what?
79. If $\sin\theta=0.3$, find the value of: $\sin\theta+\sin(\theta+2\pi)+\sin(\theta+4\pi)$.
81. If $\tan\theta=3$, find the value of: $\tan\theta+\tan(\theta+\pi)+\tan(\theta+2\pi)$.

1.9 Graphs & Transformations of the Sine and Cosine

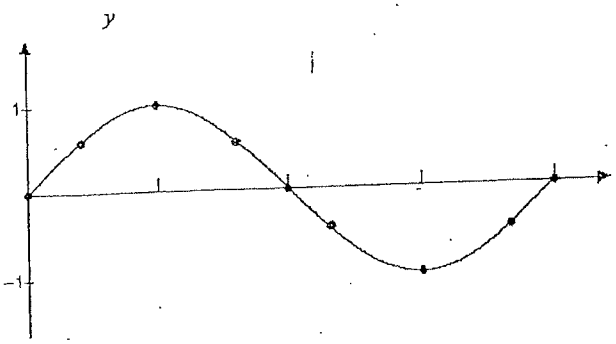
Graph of the Sine Function

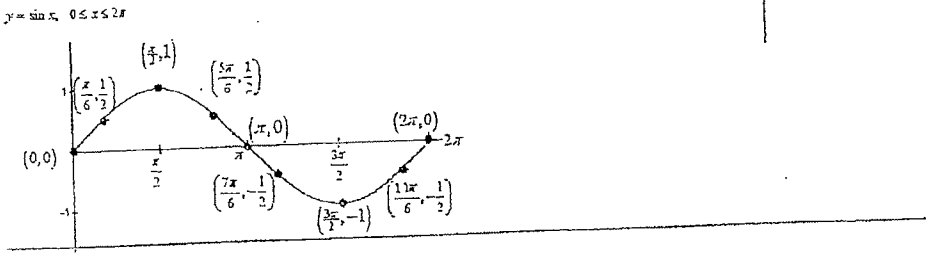
$y = \sin x, \quad -\infty \le x \le +\infty$



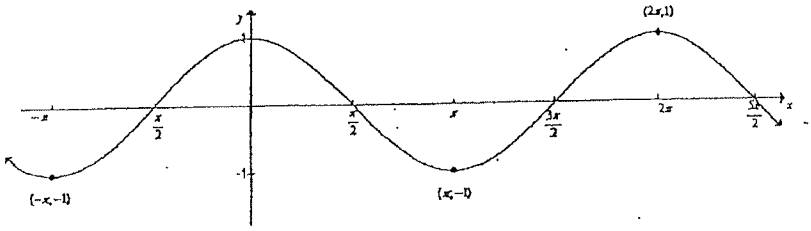
$y = \sin x, \ 0 \le x \le 2\pi$

angle "x"	sin x
0	



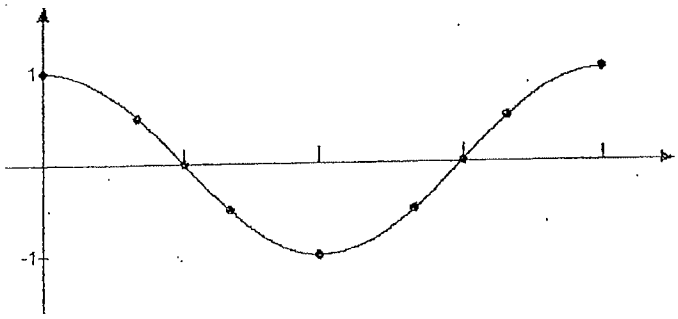


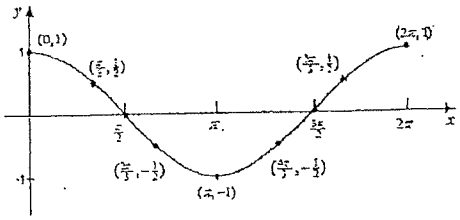
Graphs of the Cosine Function



angle "x"	cos x
0	

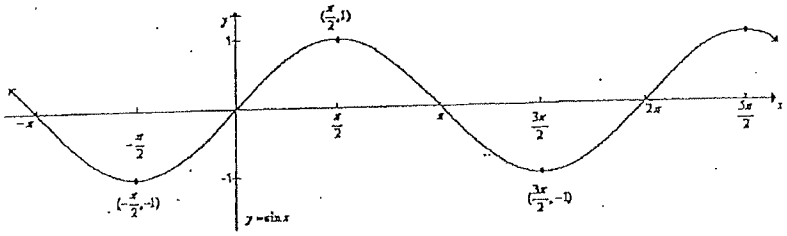
$y = \cos x, 0 \le x \le 2\pi$



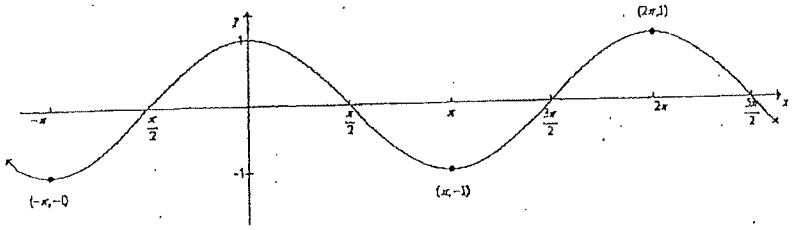


Horizontal shifting of the graph.

EXAMPLE 1 Sine shifting: Use the graph of $y = \sin x$ to graph $y = \sin\left(x - \frac{\pi}{4}\right)$.

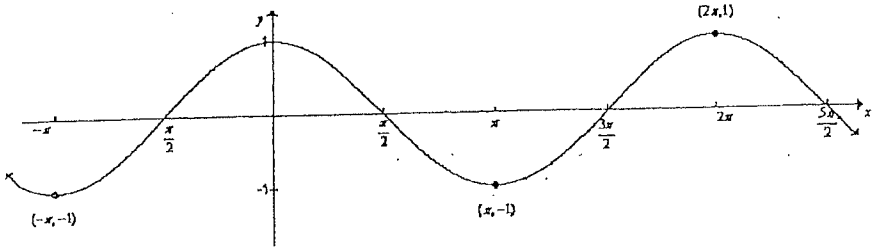


Example 3 Use the Graph of $y = \cos x$ to graph $y = 2\cos x$



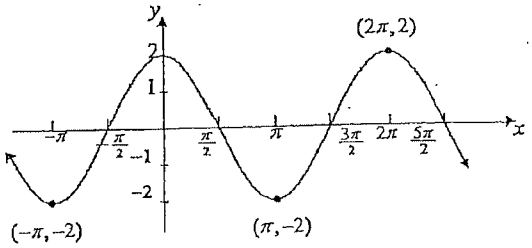
"1" is the amplitude (i.e. the largest vertical value) of the sine and cosine functions.
"2π" is the period of the sine and cosine functions.

Example Use the Graph of $y = \cos x$ to graph $y = \frac{1}{2}\cos x$.



In example 3 we obtained the graph of $y = 2\cos x$, which is reproduced below. Notice that the y-values of $y = 2\cos x$ lie between −2 and 2, inclusive.

$y = 2\cos x$



Theorem If $\omega > 0$, the amplitude and period of $y = A\sin(\omega x)$ and $y = A\cos(\omega x)$ are given by

Amplitude = $\lvert A \rvert$
Sine & Cosine Period = $\frac{2\pi}{\omega}$

$y = A\sin(\omega x - \varphi)$

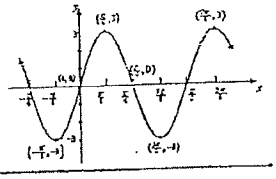
φ is the *phase shift* $|A|$ is *amplitude* ωx is the *period*

Example 5 Determine the amplitude and period of $y = 3\sin(4x)$.

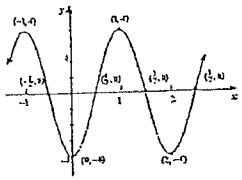
Solution: Amplitude = $|A| = 3$ and the Period = $\frac{2\pi}{\omega} = \frac{2\pi}{4} = \frac{\pi}{2}$

Example 6 Graph $y = 3\sin(4x)$

Solution:



Example 7 Determine the amplitude and period of $y = -4\cos(\pi x)$, and graph the function.



$y = A\sin(\omega x - \varphi)$ φ is the *phase shift*

$|A|$ is *amplitude* ωx is the *period*

1.9 Graphs & Transformations of the Sine and Cosine Homework

In Problems 21-35, use transformations to graph each function.

21. $y = 3\sin x$ 23. $y = -\cos x$ 25. $y = \sin x - 1$ 27. $y = \sin(x - \pi)$

29. $y = \sin(\pi x)$ 31. $y = 2\sin x + 2$ 33. $y = 4\cos(2x)$ 35. $y = -2\sin x + 2$

In Problems 37-45, determine the amplitude and period of each function without graphing.

37. $y = 2\sin x$ 39. $y = -4\cos(2x)$ 41. $y = 6\sin(\pi x)$ 43. $y = -\frac{1}{2}\cos\left(\frac{3}{2}x\right)$

45. $y = \frac{5}{3}\sin\left(-\frac{2\pi}{3}x\right)$

In Problems 61-69, graph each sinusoidal function.

61. $y = 5\sin(4x)$ 63. $y = 5\cos(\pi x)$ 65. $y = -2\cos(2\pi x) + 1$

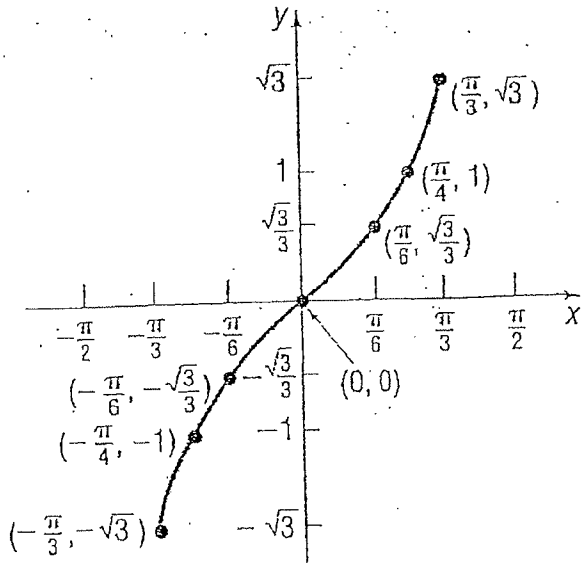
67. $y = -4\sin\left(\frac{1}{2}x\right)$ 69. $y = \frac{3}{2}\sin\left(-\frac{2}{3}x\right)$

1.10 Graphing the Tangent, Cotangent, Cosecant, Secant

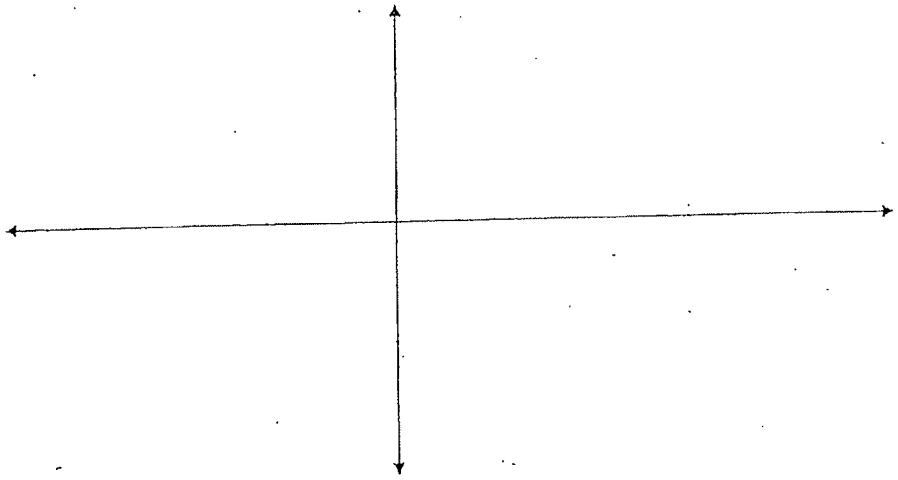
Graph $y = \tan x, \quad -\frac{\pi}{3} \le x \le \frac{\pi}{3}$

x	$y = \tan x$	(x, y)
$-\frac{\pi}{3}$	$-\sqrt{3} \approx -1.73$	$\left(-\frac{\pi}{3}, -1.73\right)$
$-\frac{\pi}{4}$	-1	$\left(-\frac{\pi}{4}, -1\right)$
$-\frac{\pi}{6}$	$-\frac{\sqrt{3}}{3} \approx -0.58$	$\left(-\frac{\pi}{6}, -.58\right)$
0	0	$(0,0)$
$\frac{\pi}{6}$	$\frac{\sqrt{3}}{3} \approx 0.58$	$\left(\frac{\pi}{6}, .58\right)$
$\frac{\pi}{4}$	1	$\left(\frac{\pi}{4}, 1\right)$
$\frac{\pi}{3}$	$\sqrt{3} \approx 1.73$	$\left(\frac{\pi}{3}, 1.73\right)$

$y = \tan x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$



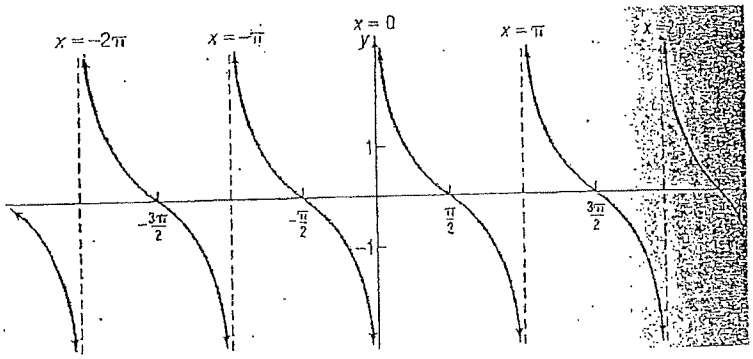
Graph $y = \tan x, \quad x = (-\infty, +\infty)$



The tangent function is discontinuous @ $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots, \frac{(2n-1)\pi}{2}$
$x = -\frac{\pi}{2}, -\frac{3\pi}{2}, -\frac{5\pi}{2}, -\frac{7\pi}{2}, \ldots, \frac{-(2n-1)\pi}{2}$

natural # n = 1,2,3,... 2n = an even number 2n-1 = an odd number

$y = \cot x$



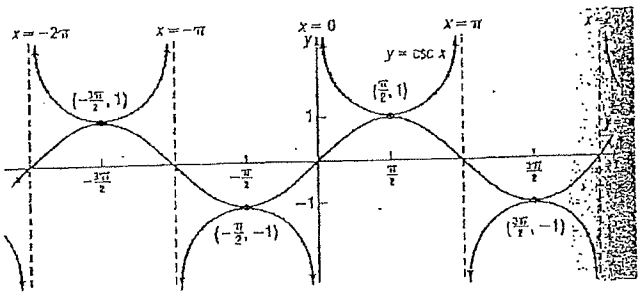
Domain =

Range =

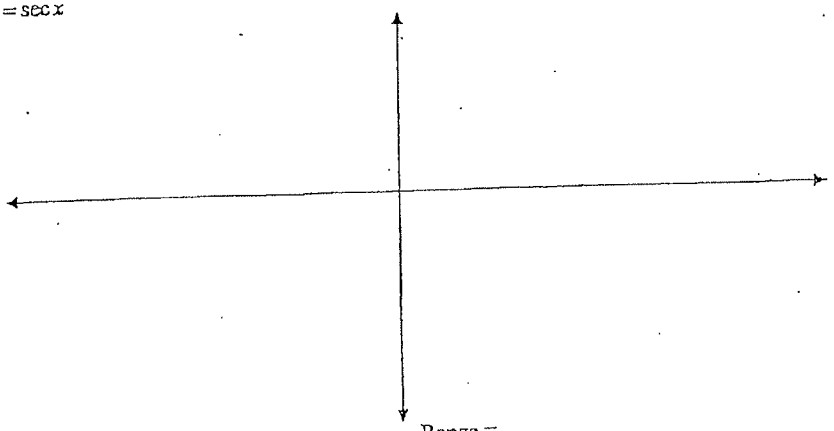
$y = \csc x$

Domain =

Range =



$y = \sec x$



Domain = Range =

1.10 Graphing the Tangent, Cotangent, Cosecant, Secant

3. The graph of $y = \tan x$ is symmetric with respect to the_____ and has vertical asymptotes at_____.

5. It is easiest to graph $y = \sec x$ by first sketching the graph of_____.

7. What is the y-intercept of $y = \tan x$?

9. What is the y-intercept of $y = \sec x$?

11. For what x-values, $-2\pi \le x \le 2\pi$, does the $\sec x = 1$? For what values of x does $\sec x = -1$?

13. For what x-values, $-2\pi \le x \le 2\pi$, does the graph of $y = \sec x$ have vertical asymptotes?

15. For what x-values, $-2\pi \le x \le 2\pi$, does the graph of $y = \tan x$ have vertical asymptotes?

In Problems 21-44, graph each function.

21. $y = -\sec x$

23. $y = \sec\left(x - \frac{\pi}{2}\right)$

25. $y = \tan(x - \pi)$

27. $y = 3\tan(2x)$

29. $y = \sec(2x)$

31. $y = \cot(\pi x)$

33. $y = -3\tan(4x)$

35. $y = 2\sec\left(\frac{1}{2}x\right)$

39. $y = \frac{1}{2}\cot\left(x - \frac{\pi}{4}\right)$

41. $y = \tan x + 2$

43. $y = \sec\left(x + \frac{\pi}{2}\right) - 1$

1.11 The Inverse Trig Functions

The Inverse Sine, Cosine, and Tangent Functions

When the angle is known, you can find the value of the trig function.

Find the sine ratio when the angle is $\frac{\pi}{3}$:.

$\sin\frac{\pi}{3}=$

When the angle is not known, you must do the inverse.

Find the angle when the sine ratio is $\frac{1}{2}$.

$\sin\theta=\frac{1}{2}$ θ is the angle whose sine ratio is $\frac{1}{2}$.

In math symbols this looks like $\theta=\arctan\frac{1}{2}$ or $\theta=\sin^{-1}\left(\frac{1}{2}\right)$

Example 1 Find the value of: $\sin^{-1}1$

Example 2 Find the value of: a) $\sin^{-1}\left(-\frac{1}{2}\right)$ b) $\cos^{-1} 0$ c) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

Solution a) $\sin^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$ b) $\cos^{-1} 0=\frac{\pi}{2}$ c) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)=\frac{3\pi}{4}$

Example 3 Find an approximate value of: (a) $\sin^{-1}\frac{1}{3}$ (b) $\sin^{-1}\left(-\frac{1}{4}\right)$

Express the answer in radians rounded to two decimal places. Use a calculator.

Solution (a) $\sin^{-1}\frac{1}{3}\approx 0.34$ (b) $\sin^{-1}\left(-\frac{1}{4}\right)\approx -0.25$

Example 6 Find the exact value of: (a) $\cos^{-1}\left[\cos\left(\frac{\pi}{12}\right)\right]$ (b) $\cos\left[\cos^{-1}(-0.4)\right]$

Solution (a) $\cos^{-1}\left[\cos\left(\frac{\pi}{12}\right)\right]=\frac{\pi}{12}$ (b) $\cos\left[\cos^{-1}(-0.4)\right]=-0.4$

Example 7 Find the exact value of: a) $\tan^{-1}1$ b) $\tan^{-1}\left(-\sqrt{3}\right)$

Example 9 Solve the equation $3\sin^{-1}x=\pi$

The solution set is $\left\{\frac{\sqrt{3}}{2}\right\}$

Graphs of Inverse Trig Functions

The Inverse Sine Function

$y = \sin^{-1} x, \quad x = [-1, +1]$ and $\theta = y = [-\frac{\pi}{2}, +\frac{\pi}{2}]$

$\theta = x = \sin y$

$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x$ when $x = [-\frac{\pi}{2}, +\frac{\pi}{2}]$

$f(f^{-1}(x)) = \sin(\sin^{-1} x) = x$ when $x = [-1, +1]$

The Inverse Cosine Function

$y = \cos^{-1} x, \quad x = [-1, +1], \quad \theta = y = [\,0, \pi\,]$

$\theta = x = \cos y$

$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x$ when $x = [0, \pi]$

$f(f^{-1}(x)) = \cos(\cos^{-1} x) = x$ when $x = [-1, +1]$

The Inverse Tangent Function $y=\tan^{-1}x, \quad x=(-\infty,+\infty), \quad \theta = y=(-\frac{\pi}{2},+\frac{\pi}{2})$

$\theta = x = \tan y$

$f^{-1}(f(x)) = \tan^{-1}(\tan x) = x \quad$ when $x=[-\frac{\pi}{2},+\frac{\pi}{2}]$

$f(f^{-1}(x)) = \tan(\tan^{-1}x) = x \quad$ when $x=(-\infty,+\infty)$

$f^{-1}(f(x)) = \sin^{-1}(\sin x) = x \quad$ when $x=[-\frac{\pi}{2},+\frac{\pi}{2}]$
$f(f^{-1}(x)) = \sin(\sin^{-1}x) = x \quad$ when $x=[-1,+1]$

$f^{-1}(f(x)) = \cos^{-1}(\cos x) = x \quad$ when $x=[0,\pi]$
$f(f^{-1}(x)) = \cos(\cos^{-1}x) = x \quad$ when $x=[-1,+1]$

$f^{-1}(f(x)) = \tan^{-1}(\tan x) = x \quad$ when $x=[-\frac{\pi}{2},+\frac{\pi}{2}]$
$f(f^{-1}(x)) = \tan(\tan^{-1}x) = x \quad$ when $x=(-\infty,+\infty)$

The Inverse Cosecant Function $y=\csc^{-1}x, \quad x\le -1$ or $x\ge +1$, and $y=\left[-\frac{\pi}{2},+\frac{\pi}{2}\right], \quad y\neq 0$

$\theta = x = \csc y$

The Inverse Secant Function $y=\sec^{-1}x$ $x\le -1$ or $x\ge +1$, and $\theta = y=[0,\ \pi]$, $y\neq\frac{\pi}{2}$

$\theta = x = \sec y$

The Inverse Cotangent Function $y=\cot^{-1}x$ $x=(-\infty,+\infty)$ and $\theta = y=(0,\ \pi)$

$\theta = x = \cot y$

Summary of Domain and Range for Inverse Trig Functions

$y=\sin^{-1}x$	$x=[-1,+1]$	and	$\theta = y=\left[-\frac{\pi}{2},+\frac{\pi}{2}\right]$
$y=\cos^{-1}x$	$x=[-1,+1]$	and	$\theta = y=[0,\ \pi]$
$y=\tan^{-1}x$	$x=(-\infty,+\infty)$	and	$\theta = y=\left(-\frac{\pi}{2},+\frac{\pi}{2}\right)$
$y=\sec^{-1}x,$	$x\le -1$ or $x\ge 1$	and	$\theta = y=[0,\ \pi],\ y\neq\frac{\pi}{2}$
$y=\csc^{-1}x$	$x\le -1$ or $x\ge +1$	and	$\theta = y=\left[-\frac{\pi}{2},+\frac{\pi}{2}\right],\ y\neq 0$
$y=\cot^{-1}x$	$x=(-\infty,+\infty)$	and	$\theta = y=(0,\ \pi)$

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1.11 The Inverse Sine, Cosine, and Tangent Functions Homework

1. What is the Domain and the range of $y=\sin x$?
2. A restriction on the domain of $f(x)=(x-1)^2$ to make it one-to-one would be $x=$_____.
3. If the domain of a one to one function is $[3,\infty)$, the range of its inverse is_____.
4. *True or False*: The graph of $y=\cos x$ is decreasing on the interval $[0,\pi]$.
5. $\tan\frac{\pi}{4}=$_____ : $\sin\frac{\pi}{3}=$_____..
6. $\sin\left(-\frac{\pi}{6}\right)=$_____; $\cos\pi=$_____;
7. $y=\sin^{-1}x$ means _____, where $-1\le x\le 1$ and $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$.
8. The value of $\sin^{-1}\left[\sin\frac{\pi}{2}\right]$ is _____.
9. $\cos^{-1}\left[\cos\frac{\pi}{5}\right]=$_____.
10. *True or False*: The domain of $y=\sin^{-1}x$ is $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$.

11 *True or False*: $\sin\left(\sin^{-1}0\right)=0$ and $\cos\left(\cos^{-1}0\right)=0$.

12 *True or False*: $y=\tan^{-1}x$ means $x=\tan y$, where $-\infty<x<\infty$ and $-\frac{\pi}{2}<y<\frac{\pi}{2}$.

In problems 13 – 23, find the exact value of each expression.

13. $\sin^{-1}0$ 15. $\sin^{-1}(-1)$ 17. $\tan^{-1}0$ 19. $\sin^{-1}\frac{\sqrt{2}}{2}$ 21. $\tan^{-1}\sqrt{3}$ 23. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

For # 25 – 35 use a calculator to find the value rounded to two decimal places.

25. $\sin^{-1}0.1$ 27. $\tan^{-1}5$ 29. $\cos^{-1}\frac{7}{8}$ 31. $\tan^{-1}(-0.4)$ 33. $\sin^{-1}(-0.12)$ 35. $\cos^{-1}\frac{\sqrt{2}}{3}$

In problems 37 – 43 find the exact value of each expression. Do not use a calculator.

37. $\sin\left[\sin^{-1}(0.54)\right]$ 39. $\cos^{-1}\left[\cos\frac{4\pi}{5}\right]$ 41. $\tan\left[\tan^{-1}(-3.5)\right]$ 43. $\sin^{-1}\left[\sin\left(-\frac{3\pi}{7}\right)\right]$

In problems 45 – 55, do not use a calculator. For your answers, also say why or why not.

45. Does $\sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right]=-\frac{\pi}{6}$?

47. Does $\sin\left[\sin^{-1}(2)\right]=2$?

49. Does $\cos^{-1}\left[\cos\left(-\frac{\pi}{6}\right)\right]=-\frac{\pi}{6}$?

51. Does $\cos\left[\cos^{-1}\left(-\frac{1}{2}\right)\right]=-\frac{1}{2}$?

53. Does $\tan^{-1}\left[\tan\left(-\frac{\pi}{3}\right)\right]=-\frac{\pi}{3}$?

55. Does $\tan\left[\tan^{-1}(2)\right]=2$?

In problems 57 – 63, find the exact solution of each equation.

57. $4\sin^{-1}(x)=\pi$ 59. $3\cos^{-1}(2x)=2\pi$ 61. $3\tan^{-1}x=\pi$ 63. $4\cos^{-1}x-2\pi=2\cos^{-1}x$

1.12 Values of Inverse Trig Functions

Find the exact value of $\sin^{-1}(\sin\frac{5\pi}{4})$.

Find the exact value of $\sin(\tan^{-1}\frac{1}{2})$.

Find the exact value of $\cos(\sin^{-1}\frac{1}{2})$

Find the exact value of $\tan(\cos^{-1}(-\frac{1}{3}))$

Use your calculator to approximate the value in radians to two decimal places:

a) $\sec^{-1}3$

b) $\csc^{-1}(-4)$

c) $\cot^{-1}\left(\frac{1}{2}\right)$

d) $\cot^{-1}(-2)$

1.12 Values of Inverse Trig Functions Homework

1. What is the domain and the range of $y=\sec x$?
2. Is the graph of $y=\sec x$ increasing on the intervals $\left[0,\frac{\pi}{2}\right)$ and $\left[\frac{\pi}{2},\pi\right)$?
3. If $\cot\theta=-2$ and $0<\theta<\pi$, then $\cos\theta=$_____.
4. $y=\sec^{-1}x$ means _____, where $|x|$_____ *and* _____ $\le y\le$ _____, $y\neq\frac{\pi}{2}$.
5. $\cos\left(\tan^{-1}1\right)=$_____.
6. *True or false*: You cannot obtain exact values for the inverse secant function.
7. *True or false*: $\csc^{-1}0.5$ is not defined.
8. *True or false*: The domain of the inverse cotangent function is the set of real numbers.

In problems 9 – 35, find the value of each expression.

9. $\cos\left(\sin^{-1}\frac{\sqrt{2}}{2}\right)$
11. $\tan\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$
13. $\sec\left(\cos^{-1}\frac{1}{2}\right)$
15. $\csc\left(\tan^{-1}1\right)$
17. $\sin\left[\tan^{-1}(-1)\right]$
19. $\sec\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$
21. $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$
23. $\sin^{-1}\left[\sin\left(-\frac{7\pi}{6}\right)\right]$
25. $\tan\left(\sin^{-1}\frac{1}{3}\right)$
27. $\sec\left(\tan^{-1}\frac{1}{2}\right)$
29. $\cot\left[\sin^{-1}\left(-\frac{\sqrt{2}}{3}\right)\right]$
31. $\sin\left[\tan^{-1}(-3)\right]$
33. $\sec\left(\sin^{-1}\frac{2\sqrt{5}}{5}\right)$
35. $\sin^{-1}\left(\cos\frac{3\pi}{4}\right)$

In problems 37 – 43, find the exact value of each expression.

37. $\cot^{-1}\sqrt{3}$
39. $\csc^{-1}(-1)$
41. $\sec^{-1}\frac{2\sqrt{3}}{3}$
43. $\cot^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

Use a calculator to find the value of each expression rounded to two decimal places.

45. $\sec^{-1}4$
47. $\cot^{-1}2$
49. $\csc^{-1}(-3)$
51. $\cot^{-1}\left(-\sqrt{5}\right)$
53. $\csc^{-1}\left(-\frac{3}{2}\right)$
55. $\cot^{-1}\left(-\frac{3}{2}\right)$

$f(x)=\sin x$, and $h(x)=\tan x$. Find the exact value of each composite function.

57. $g\left(f^{-1}\left(\frac{12}{13}\right)\right)$
59. $g^{-1}\left(f\left(\frac{7\pi}{4}\right)\right)$
61. $h\left(f^{-1}\left(-\frac{3}{5}\right)\right)$
63. $g\left(h^{-1}\left(\frac{12}{5}\right)\right)$
65. $g^{-1}\left(f\left(-\frac{4\pi}{3}\right)\right)$
67. $h\left(g^{-1}\left(-\frac{1}{4}\right)\right)$

1.13 Trigonometric Identities

Identities and proofs, addition formulas, double-angle, half-angle, reduction formulas
Methods used to Simplify Trigonometric Expressions

Example 1

a) Simplify $\frac{\cot\theta}{\csc\theta}$ by rewriting in terms of sine and cosine functions.

b) Show that $\frac{\cos\theta}{1+\sin\theta}=\frac{1-\sin}{\cos\theta}$ by multiplying the numerator & denominator by conjugate of the denominator i.e. $1-\sin\theta$.

c) Simplify $\frac{1+\sin\theta}{\sin\theta}+\frac{\cot\theta-\cos\theta}{\cos\theta}$ by adding two fractions i.e. getting a common denominator

d) Simplify $\frac{\sin^2\theta-1}{\tan\theta\sin\theta-\tan\theta}$ by factoring.

a) $\frac{\cot\theta}{\csc\theta}$

Solution: a) $\frac{\cot\theta}{\csc\theta}=\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}=\frac{\cos\theta}{\sin\theta}\cdot\frac{\sin\theta}{1}=\cos\theta$

b) $\frac{\cos\theta}{1+\sin\theta}\Rightarrow\frac{\cos\theta}{1+\sin\theta}\cdot\frac{1-\sin\theta}{1-\sin\theta}\Rightarrow\frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}\Rightarrow\frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}\Rightarrow\frac{1-\sin\theta}{\cos\theta}$

c) $\frac{1+\sin\theta}{\sin\theta}+\frac{\cot\theta-\cos\theta}{\cos\theta}\Rightarrow\frac{1+\sin\theta}{\sin\theta}\cdot\frac{\cos\theta}{\cos\theta}+\frac{\cot\theta-\cos\theta}{\cos\theta}\cdot\frac{\sin\theta}{\sin\theta}\Rightarrow\frac{\cos\theta+\sin\theta\cos\theta+\cot\theta\sin\theta}{\sin\theta\cos\theta}$

$\Rightarrow\frac{\cos\theta+\frac{\cos\theta}{\sin\theta}\cdot\sin\theta}{\sin\theta\cos\theta}\Rightarrow\frac{\cos\theta+\cos\theta}{\sin\theta\cos\theta}\Rightarrow\frac{2\cos\theta}{\sin\theta\cos\theta}\Rightarrow\frac{2}{\sin\theta}$

d) $\frac{\sin^2\theta-1}{\tan\theta\sin\theta-\tan\theta}\Rightarrow\frac{(\sin\theta+1)(\sin\theta-1)}{\tan\theta(\sin\theta-1)}\Rightarrow\frac{\sin\theta+1}{\tan\theta}$

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Example 2 Verify that $\csc\theta \cdot \tan\theta = \sec\theta$

Hint: Start with the left side, because it contains the more complicated expression.

Example 3 Verify that $\sin^2(-\theta) + \cos^2(-\theta) = 1$

Example 4 Verify $\dfrac{\sin^2(-\theta) - \cos^2(-\theta)}{\sin(-\theta) - \cos(-\theta)} = \cos\theta - \sin\theta$

Example 5 Verify that $\dfrac{1+\tan\theta}{1+\cot\theta} = \tan\theta$

Example 6 Verify that $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2\csc\theta$

Example 7 Verify that $\frac{\tan\theta+\cot\theta}{\sec\theta\csc\theta}=1$

Example 8 Verify that $\frac{1-\sin\theta}{\cos\theta}=\frac{\cos\theta}{1+\sin\theta}$

Guidelines for Establishing Identities

1. It is usually best to start with the more complicated expression.
2. Add or subtract fractions.
3. Rewrite one side in terms of sine and cosine functions only.
4. Always keep in mind the form of the expression on the other side.

1.13 Trigonometric Identities Homework

1. *True or False*: $\sin^2\theta = 1 - \cos^{-2}\theta$
2. *True or False*: $\sin(-\theta) + \cos(-\theta) = \cos\theta - \sin\theta$.
4. $\tan^2\theta - \sec^2\theta$.
5. $\cos(-\theta) - \cos\theta =$ _____.
6. *True or False*: $\sin(-\theta) + \sin\theta = 0$ for any value of θ.

Simplify problems 9 – 17 by following the indicated direction.

9. Rewrite in terms of sine and cosine functions: $\tan\theta\csc\theta$
11. Multiply $\frac{\cos\theta}{1-\sin\theta}$ *by* $\frac{1+\sin\theta}{1+\sin\theta}$
13. Rewrite over a common denominator: $\frac{\sin\theta+\cos\theta}{\cos\theta} + \frac{\cos\theta-\sin\theta}{\sin\theta}$
15. Multiply and simplify: $\frac{(\sin\theta+\cos\theta)(\sin\theta+\cos\theta)-1}{\sin\theta\cos\theta}$
17. Factor and simplify: $\frac{3\sin^2\theta+4\sin\theta+1}{\sin^2\theta+2\sin\theta+1}$

In problems 19 – 103, establish each identity.

19. $\csc\theta\cos\theta = \cot\theta$
21. $1+\tan^2(-\theta) = \sec^2\theta$
23. $\cos\theta(\tan\theta+\cot\theta) = \csc\theta$
25. $\tan\theta\cot\theta - \cos^2\theta = \sin^2\theta$
27. $(\sec\theta-1)(\sec\theta+1) = \tan^2\theta$
29. $(\sec\theta+\tan\theta)(\sec\theta-\tan\theta) = 1$
31. $\cos^2\theta(1+\tan^2\theta) = 1$
33. $(\sin\theta+\cos\theta)^2 + (\sin\theta-\cos\theta)^2 = 2$
35. $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$
37. $\sec\theta - \tan\theta = \frac{\cos\theta}{1+\sin\theta}$
39. $3\sin^2\theta + 4\cos^2\theta = 3 + \cos^2\theta$
41. $1 - \frac{\cos^2}{1+\sin\theta} = \sin\theta$
43. $\frac{1+\tan\theta}{1-\tan\theta} = \frac{\cot\theta+1}{\cot\theta-1}$
45. $\frac{\sec\theta}{\csc\theta} + \frac{\sin\theta}{\cos\theta} = 2\tan\theta$
47. $\frac{1+\sin\theta}{1-\sin\theta} = \frac{\csc\theta+1}{\csc\theta-1}$
49. $\frac{1-\sin\theta}{\cos\theta} + \frac{\cos\theta}{1-\sin\theta} = 2\sec\theta$
51. $\frac{\sin\theta}{\sin\theta-\cos\theta} = \frac{1}{1-\cot\theta}$
53. $\frac{1-\sin\theta}{1+\sin\theta} = (\sec\theta-\tan\theta)^2$

55. $\dfrac{\cos\theta}{1-\tan\theta}+\dfrac{\sin\theta}{1-\cot\theta}=\sin\theta+\cos\theta$ 57. $\tan\theta+\dfrac{\cos\theta}{1+\sin\theta}=\sec\theta$

1.14 The Sum and Difference Formulas

Sum and Difference Formulas for the Cosine Function

The cosine of the *sum* of two angles equals the cosine of the first angle times the cosine of the second angle *minus* the sine of the first angle times the sine of the second angle.

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

Memorize these:

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

Example 1 Find the exact value of $\cos 75^\circ$. Hint: 75=45+30

Solution: $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

Example 2 Find the exact value of $\cos\dfrac{\pi}{12}$. Hint: $\frac{\pi}{12}=\frac{\pi}{4}-\frac{\pi}{6}$

Solution: $\dfrac{1}{4}\left(\sqrt{6}+\sqrt{2}\right)$

Verify the cofunction formula: $\cos\left(\dfrac{\pi}{2}-\theta\right)=\sin\theta$ Cofunctions of complementary angles are equal.

$\cos\left(\dfrac{\pi}{2}-\theta\right)=\cos\dfrac{\pi}{2}\cos\theta\sin\dfrac{\pi}{2}\sin\theta$ = 0□cos θ + 1□sin θ = sin θ

Verify the cofunction formula: $\sin\left(\frac{\pi}{2}-\theta\right)$ = $\cos\theta$

Using the previous cosine cofunction formula i.e. $\left\{\cos\left(\dfrac{\pi}{2}-\theta\right)=\sin\theta\right\}$ in reverse gives us

$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\left[\frac{\pi}{2}-\left(\frac{\pi}{2}-\theta\right)\right] = \cos\theta$ thus establishing the cofunction formula.

Sum and Difference Formulas for the Sine Function

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$

Using the cofunction formula for cosine in reverse gives us

$$\begin{aligned}\sin(\alpha+\beta) &= \cos\left(\tfrac{\pi}{2}-(\alpha+\beta)\right) = \cos\left(\left(\tfrac{\pi}{2}-\alpha\right)-\beta\right)\\ &= \cos\left(\tfrac{\pi}{2}-\alpha\right)\cos\beta+\sin\left(\tfrac{\pi}{2}-\alpha\right)\sin\beta\\ &= \sin\alpha\ \cos\beta\ +\ \cos\alpha\ \sin\beta\end{aligned}$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta+\cos\alpha\sin\beta$$

Using the sum formula for sine gives us

$$\begin{aligned}\sin(\alpha-\beta) &= \sin[\ \alpha\ +\ (-\beta)\ ]\\ &= \sin\alpha\cos(-\beta)+\cos\alpha\sin(-\beta)\\ &= \sin\alpha\cos\beta+\cos\alpha(-\sin\beta)\end{aligned}$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta-\cos\alpha\sin\beta$$

Example 3 Find the exact value of $\sin\frac{7\pi}{12}$.

Solution $=\frac{1}{4}\left(\sqrt{2}+\sqrt{6}\right)$

Example 4 Find the exact value of $\sin 80^\circ\cos 20^\circ-\cos 80^\circ\sin 20^\circ$

Solution $= \sin 60^\circ = \frac{\sqrt{3}}{2}$

Example 5 If $\sin\alpha = \frac{4}{5}$, $\alpha = \left(\frac{\pi}{2}, \pi\right)$ and $\sin\beta = -\frac{2}{\sqrt{5}}$, $\beta = \left(\pi, \frac{3\pi}{2}\right)$

find the exact value of (a) $\cos\alpha$ (b) $\cos\beta$

(c) $\cos(\alpha+\beta)$ (d) $\sin(\alpha+\beta)$

Solution: (a) $\cos\alpha = -\frac{3}{5}$ (b) $\cos\beta = -\frac{\sqrt{5}}{5}$ (c) $\cos(\alpha+\beta) = \frac{11\sqrt{5}}{25}$ (d) $\sin(\alpha+\beta) = \frac{2\sqrt{5}}{25}$

Example 6 Verify that $\frac{\cos(\alpha-\beta)}{\sin\alpha\sin\beta} = \cot\alpha\cot\beta + 1$

Example 7 Prove that $\tan(\theta+\pi)=\tan\theta$. This verifies that the tangent has a period of π.

Example 8 Prove the identity: $\tan\left(\theta+\frac{\pi}{2}\right)=-\cot\theta$

Proof of the sum and difference formulas for the tangent function

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

Proof:

$$\tan(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$$

divide the numerator and the denominator by $\cos\alpha\cos\beta$

$$\tan(\alpha+\beta)=\frac{\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}=\frac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}-\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$\tan(\alpha+\beta)=\frac{\frac{\sin\alpha}{\cos\alpha}+\frac{\sin\beta}{\cos\beta}}{\frac{\cos\beta}{\cos\beta}-\frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$ Q.E.D.

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Proof:

$$\tan(\alpha-\beta)=\tan[\alpha+(-\beta)]$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha+\tan(-\beta)}{1-\tan\alpha\tan(-\beta)}$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan(\beta)}{1-\tan\alpha(-\tan\beta)}$$

$\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$ Q.E.D.

1.14 The Sum and Difference Formulas Homework

1. The distance d from the point $(2,-3)$ to the point $(5,1)$ is _____.

2. If $\sin\theta=\frac{4}{5}$ and θ is in quadrant II, then $\cos\theta=$ _____.

3. (a) $\sin\frac{\pi}{4}\cos\frac{\pi}{3}=$ _____. (b) $\tan\frac{\pi}{4}-\sin\frac{\pi}{6}=$ _____.

4. $\cos(\alpha+\beta)=\cos\alpha\cos\beta$ _____ $\sin\alpha\sin\beta$

5. $\sin(\alpha-\beta)=\sin\alpha\cos\beta$ _____ $\cos\alpha\sin\beta$

6. *True or False:* $\sin(\alpha+\beta)=\sin\alpha+\sin\beta+2\sin\alpha\sin\beta$

7. *True or False:* $\tan 75^\circ=\tan 30^\circ+\tan 45^\circ$

8. *True or False:* $\cos\left(\frac{\pi}{2}-\theta\right)=\cos\theta$

In problems 9 – 19, find the exact value of each expression.

9. $\sin\frac{5\pi}{12}$ 11. $\cos\frac{7\pi}{12}$ 13. $\cos 165^\circ$ 15. $\tan 15^\circ$ 17. $\sin\frac{17\pi}{12}$ 19. $\sec\left(-\frac{\pi}{12}\right)$

In problems 21 – 29, find the exact value of each expression.

21. $\sin 20^\circ\cos 10^\circ+\cos 20^\circ\sin 10^\circ$ 23. $\cos 70^\circ\cos 20^\circ-\sin 70^\circ\sin 20^\circ$

25. $\dfrac{\tan 20^\circ+\tan 25^\circ}{1-\tan 20^\circ\tan 25^\circ}$ 27. $\sin\frac{\pi}{12}\cos\frac{7\pi}{12}-\cos\frac{\pi}{12}\sin\frac{7\pi}{12}$ 29. $\cos\frac{\pi}{12}\cos\frac{5\pi}{12}+\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

Find the exact value of problems 31-35 under the given conditions:

(a) $\sin(\alpha+\beta)$ (b) $\cos(\alpha+\beta)$ (c) $\sin(\alpha-\beta)$ (d) $\tan(\alpha-\beta)$

31. $\sin\alpha=\frac{3}{5}$, $0<\alpha<\frac{\pi}{2}$; $\cos\beta=\frac{2\sqrt{5}}{5}$, $-\frac{\pi}{2}<\beta<0$

33. $\tan\alpha=-\frac{4}{3}$, $\frac{\pi}{2}<\alpha<\pi$; $\cos\beta=\frac{1}{2}$, $0<\beta<\frac{\pi}{2}$

35. $\sin\alpha=\frac{5}{13}$, $-\frac{3\pi}{2}<\alpha<-\pi$; $\tan\beta=-\sqrt{3}$, $\frac{\pi}{2}<\beta<\pi$

37. If $\sin\theta=\frac{1}{3}$, θ is in quadrant 2, find the exact value of:

(a) $\cos\theta$ (b) $\sin\left(\theta+\frac{\pi}{6}\right)$ (c) $\cos\left(\theta-\frac{\pi}{3}\right)$ (d) $\tan\left(\theta+\frac{\pi}{4}\right)$

In problems 45 –55, establish each identity. 45. $\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$

47. $\sin(\pi-\theta)=\sin\theta$ 49. $\sin(\pi+\theta)=-\sin\theta$ 51. $\tan(\pi-\theta)=-\tan\theta$

53. $\sin\left(\frac{3\pi}{2}+\theta\right)=-\cos\theta$ 55. $\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta$

In problems 71 & 73, find the exact value of each expression.

71. $\sin\left(\sin^{-1}\frac{1}{2}+\cos^{-1}0\right)$

73. $\sin\left[\sin^{-1}\frac{3}{5}-\cos^{-1}\left(-\frac{4}{5}\right)\right]$

1.15 Double-angle and Power Reducing Formulas

Double-angle Formulas

$\sin(2\theta)=2\sin\theta\cos\theta$

$\cos(2\theta)=\cos^2\theta-\sin^2\theta$

$\cos(2\theta)=1-2\sin^2\theta$

$\cos(2\theta)=2\cos^2\theta-1$

$\tan(2\theta)=\frac{2\tan\theta}{1-\tan^2\theta}$

Power Reducing Formulas

$\sin^2\theta=\frac{1-\cos(2\theta)}{2}$

$\cos^2\theta=\frac{1+\cos(2\theta)}{2}$

$\tan^2\theta=\frac{1-\cos(2\theta)}{1+\cos(2\theta)}$

Derive the formula for $\sin(2\theta)$

Derive the formulas for $\cos(2\theta)$

Derive the formula for $\tan(2\theta)$

If $\sin\theta = \frac{3}{5}$, $\theta = \left(\frac{\pi}{2}, \pi\right)$, find

a) $\sin(2\theta)$

b) $\cos(2\theta)$

Derive the formula for $\sin^2\theta$

Derive the formula for $\cos^2\theta$

Derive the formula for $\tan^2\theta$

Rewrite $\cos^4\theta$ without any powers of sine or cosine greater than 1.

1.15 Double-angle and Power-reducing Formulas Homework

1. $\cos(2\theta)=\cos^2\theta-$_____$=$_____$-1=1-$_____.

2. $\sin^2\frac{\theta}{2}=\frac{\quad}{2}$.

3. $\tan\frac{\theta}{2}=\frac{1-\cos\theta}{\quad}$.

4. *Does* $\cos(2\theta)$ have these equivalent forms: $\cos^2\theta-\sin^2\theta$, $1-2\sin^2\theta$, $2\cos^2\theta-1$?

5. *Does* $\sin(2\theta)$ have these equivalent forms: $2\sin\theta\cos\theta$ and $\sin^2\theta-\cos^2\theta$?

6. *Does* $\tan(2\theta)+\tan(2\theta)=\tan(4\theta)$

In problems 7 - 17, $0\le\theta<2\pi$. Use the information given to find the exact value of:

(a) $\sin(2\theta)$ (b) $\cos(2\theta)$ (c) $\sin\frac{\theta}{2}$ (d) $\cos\frac{\theta}{2}$

7. $\sin\theta=\frac{3}{5},0<\theta<\frac{\pi}{2}$ 9. $\tan\theta=\frac{4}{3},\pi<\theta<\frac{3\pi}{2}$ 11. $\cos\theta=-\frac{\sqrt{6}}{3},\frac{\pi}{2}<\theta<\pi$

13. $\sec\theta=3,\sin\theta>0$ 15. $\cot\theta=-2,\sec\theta<0$ 17. $\tan\theta=-3,\sin\theta<0$

Use Power Reducing Formulas to find the exact value of #19-27.

19. $\sin 22.5^\circ$ 21. $\tan\frac{7\pi}{8}$ 23. $\cos 165^\circ$ 25. $\sec\frac{15\pi}{8}$ 27. $\sin\left(-\frac{\pi}{8}\right)$

41. Show that $\sin^4\theta=\frac{3}{8}-\frac{1}{2}\cos(2\theta)+\frac{1}{8}\cos(4\theta)$

In problems 47 - 67, establish each identity.

47. $\cos^4\theta-\sin^4\theta=\cos(2\theta)$ 49. $\cot(2\theta)=\frac{\cot^2\theta-1}{2\cot\theta}$ 51. $\sec(2\theta)=\frac{\sec^2\theta}{2-\sec^2\theta}$

53. $\cos^2(2\theta)-\sin^2(2\theta)=\cos(4\theta)$ 55. $\frac{\cos(2\theta)}{1+\sin(2\theta)}=\frac{\cot\theta-1}{\cot\theta+1}$ 57. $\sec^2\frac{\theta}{2}=\frac{2}{1+\cos\theta}$

59. $\cot^2\frac{\theta}{2}=\frac{\sec\theta+1}{\sec\theta-1}$ 61. $\cos\theta=\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$ 63. $\frac{\sin(3\theta)}{\sin\theta}-\frac{\cos(3\theta)}{\cos}=2$

65. $\tan(3\theta)=\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$ 67. $\ln|\sin\theta|=\frac{1}{2}\left(\ln|1-\cos(2\theta)|-\ln 2\right)$

In problems 69 - 79, find the exact value of each expression.

69. $\sin\left(2\sin^{-1}\frac{1}{2}\right)$ 71. $\cos\left(2\sin^{-1}\frac{3}{5}\right)$ 73. $\tan\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right]$

75. $\sin\left(2\cos^{-1}\frac{4}{5}\right)$ 77. $\sin^2\left(\frac{1}{2}\cos^{-1}\frac{3}{5}\right)$ 79. $\sec\left(2\tan^{-1}\frac{3}{4}\right)$

1.16 Solving Trigonometric Equations

Example 1: Is $\theta=\frac{\pi}{4}$ a solution to the equation $\sin\theta=\frac{1}{2}$? If not, what is the solution?

Example 2: Solve the equation $\cos\theta=\frac{1}{2}$.

Solution: $\theta=\frac{\pi}{3}+2k\pi$, $\theta=\frac{5\pi}{3}+2k\pi$ where k is an integer

Example 3: Solve the equation $2\sin\theta+\sqrt{3}=0$, $0\le\theta<2\pi$.

Hint: Solve for sine. Solution: $x=\frac{4\pi}{3},\frac{5\pi}{3}$ or 240°, 300°

Example 4: Solve the equation $\sin 2\theta=\frac{1}{2}$ where $\theta=[0,2\pi)$.

Solution: $\theta=\frac{\pi}{12}+k\pi,\ \theta=\frac{5\pi}{12}+k\pi,\ k=0,1,2\Rightarrow\theta=\left\{\frac{\pi}{12},\frac{5\pi}{12},\frac{13\pi}{12},\frac{17\pi}{12}\right\}$

Example 1 Solve the quadratic trinomial equation: $2\sin^2\theta-3\sin\theta+1=0,\ 0\le\theta<2\pi$

Hint: factor. The solution set is $\left\{\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}\right\}$.

Example 2 Solve the equation using identities: $3\cos\theta + 3 = 2\sin^2\theta, \quad 0 \le \theta < 2\pi$

Hint: Use Pythagorean Identity $\sin^2\theta = \ldots$ The solution set is $\left\{\frac{2\pi}{3}, \pi, \frac{4\pi}{3}\right\}$.

Example 3 Solve the equation: $\cos(2\theta) + 3 = 5\cos\theta, \quad 0 \le \theta < 2\pi$

Hint: Reduce 2θ. Use double angle formula $\cos 2\theta = 2\cos^2\theta - 1$ The solution set is $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$.

Example 4 Solve the equation: $\cos^2\theta + \sin\theta = 2, \quad 0 \le \theta < 2\pi$

Hint: Use Pythagorean Identity to write in terms of $\sin\theta$. Solution: $\sin^2\theta-\sin\theta+1=0$

1.16 Solving Trigonometric Equations

In # 7 - 29, solve each equation on the interval $0\le\theta<2\pi$.

7. $2\sin\theta+3=2$
9. $4\cos^2\theta=1$
11. $2\sin^2\theta-1=0$
13. $\sin(3\theta)=-1$
15. $\cos(2\theta)=-\frac{1}{2}$
17. $\sec\frac{3\theta}{2}=-2$
19. $2\sin\theta+1=0$
21. $\tan\theta+1=0$
23. $4\sec\theta+6=-2$
25. $3\sqrt{2}\cos\theta+2=-1$
27. $\cos\left(2\theta-\frac{\pi}{2}\right)=-1$
29. $\tan\left(\frac{\theta}{2}+\frac{\pi}{3}\right)=1$

Solve #31–39. Give a general formula for all the solutions. List six solutions.

31. $\sin\theta=\frac{1}{2}$
33. $\tan\theta=-\frac{\sqrt{3}}{3}$
35. $\cos\theta=0$
37. $\cos(2\theta)=-\frac{1}{2}$
39. $\sin\frac{\theta}{2}=-\frac{\sqrt{3}}{2}$

Solve #41 – 51 for $\theta=[0,2\pi)$. Round answers to two decimal places.

41. $\sin\theta=0.4$
43. $\tan\theta=5$
45. $\cos\theta=-0.9$
47. $\sec\theta=-4$
49. $5\tan\theta+9=0$
51. $3\sin\theta-2=0$

---

1. Find the real solutions of $4x^2-x-5=0$.
2. Find the real solutions of $x^2-x-1=0$.
3. Find the real solutions of $(2x-1)^2-3(2x-1)-4=0$.
4. Use a graphing utility to solve $5x^3-2=x-x^2$. Round answers to two decimal places.

In problems 5 – 45, solve each equation on the interval $0\le\theta<2\pi$.

5. $2\cos^2\theta+\cos\theta=0$
7. $2\sin^2\theta-\sin\theta-1=0$
9. $(\tan\theta-1)(\sec\theta-1)=0$
11. $\sin^2\theta-\cos^2\theta=1+\cos\theta$
13. $\sin^2\theta=6(\cos\theta+1)$
15. $\cos(2\theta)+6\sin^2\theta=4$
17. $\cos\theta=\sin\theta$
19. $\tan\theta=2\sin\theta$
21. $\sin\theta=\csc\theta$
23. $\cos(2\theta)=\cos\theta$
25. $\sin(2\theta)+\sin(4\theta)=0$
27. $\cos(4\theta)-\cos(6\theta)=0$
29. $1+\sin\theta=2\cos^2\theta$
31. $2\sin^2\theta-5\sin\theta+3=0$
33. $3(1-\cos\theta)=\sin^2\theta$
35. $\tan^2\theta=\frac{3}{2}\sec\theta$
37. $3-\sin\theta=\cos(2\theta)$
41. $\sin\theta-\sqrt{3}\cos\theta=1$
43. $\tan(2\theta)+2\sin\theta=0$
45. $\sin\theta+\cos\theta=\sqrt{2}$

In # 47 - 51, find the real zeros of each trig function on the interval $0\le x<2\pi$.

47. $f(x)=4\cos^2x-1$
49. $f(x)=\sin(2x)-\sin x$
51. $\sin x+\cos x=x$

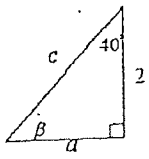
Solve #53 – 63 with a calculator. Round solution(s) to two decimal places.

53. $x+5\cos x=0$
55. $22x-17\sin x=3$
57. $\sin x+\cos x=x$

59. $x^2 = x + 3\cos(2x)$ 61. $x^2 - 2\sin(2x) = 3x$ 63. $6\sin x - e^x = 2, x > 0$

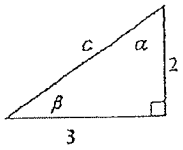
1.17 Applications Involving Right Triangles

Example 1: If $b=2$ and $\alpha = 40^\circ$, find a, c, and β in the right triangle.



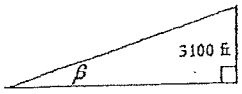
Solution: $a = 2\tan 40^\circ \approx 1.68$ and $c = \dfrac{2}{\cos 40^\circ} \approx 2.61$, $\beta = 50^\circ$.

Example 2: If If $a=3$ and $b=2$, find c, α, and β.



Solution: $c=\sqrt{13}\approx 3.61,\ \alpha=\tan^{-1}\frac{3}{2},\ \beta=33.7^{\circ}$

Example 3: A straight trail leads from the Alpine Hotel, elevation 8000 feet, to a scenic overlook, elevation 11,100 feet. The length of the trail is 14,100 feet. What is the inclination (grade) of the trail? That is, what is the angle β in Figure 4?



Solution: $\beta=\sin^{-1}\frac{3100}{14{,}100}\approx 12.7^{\circ}$ The inclination (grade) of the trail is approximately 12.7°

1.17 Applications Involving Right Triangles Homework

1. In a right triangle, if the length of the hypotenuse is 5 and the length of one of the other sides is 3, what is the length of the third side?
2. *True or False*: The angles 52° and 48° are complementary.
3. If θ is an acute, solve the equation $\tan\theta=\frac{1}{2}$. Round answer to one decimal place.
4. If θ is an acute angle, solve the equation $\sin\theta=\frac{1}{2}$.
5. *True or False*: In a right triangle, one of the angles is 90° and the sum of the other two angles is 90°.
6. In navigation or surveying, the _____ from a point O to a point P equals the acute angle θ between ray OP and the vertical line through O, the north – south line.
7. *True or False*: In a right triangle, if two sides are known, we can solve the triangle.
8. *True or False*: In a right triangle, if we know the two acute angles, we can solve the triangle.

In problems 9 – 21, using the given information, solve the right triangle.

9. $b=5,\ \beta=20^{\circ}$; find a, c, and α
11. $a=6,\ \beta=40^{\circ}$; find b, c, and α
13. $b=4,\ \alpha=10^{\circ}$; find a, c, and β
15. $a=5,\ \alpha=25^{\circ}$; find b, c, and β
17. $c=9,\ \beta=20$; find a, c, and α
19. $a=5,\ b=3$; find c, α, and β
21. $a=2, c=5$; find b, α, and β
23. The hypotenuse of a right triangle is 5 inches. If one leg is 2 inches, find the degree measure of each angle.

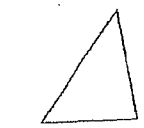
24. The hypotenuse of a right triangle is 3 feet. If one leg is 1 foot, find the degree measure of each angle.

25. At 10 am on April 26, 2005, a building 300 feet high casts a shadow 50 feet long. What is the angle of elevation of the Sun?

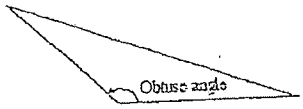
1.18 Law of Sines

If none of the angles of a triangle is a right angle, the triangle is called **oblique**.

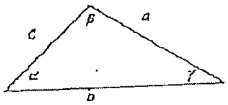
An oblique triangle will have either three acute angles
or two acute angles and one obtuse angle (an angle between 90° and 180°).



(a) All angles are acute



(b) Two acute angles and one obtuse angle

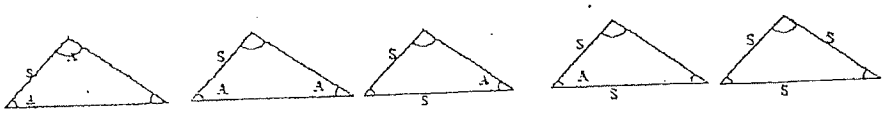


In the discussion that follows, we will always label an oblique triangle so that a is opposite angle α, side b is opposite angle β, and side c is opposite angle γ.

To solve an oblique triangle means to find the lengths of its sides and the measurements of its angles. To do this, we shall need to know the length of one side* along with (i) two angles; (ii) one angle and one other side; (iii) the other two sides. There are four possibilities to consider:

CASE 1: One side and two angles are known (ASA or SAA).
CASE 2: Two sides and the angle opposite one of them are known (SSA).
CASE 3: Two sides and the included angle are known (SAS).
CASE 4: Three are known (SSS).

The four cases are illustrated below.



Case 1: ASA Case 1: SAA Case 2: SSA Case 3: SAS Case 4 : SSS

Use the Law of Sines to solve triangles for Case 1 or Case 2.

Use the Law of Cosines to solve triangles for Case 3 or Case 4.

Law of Sines

For a triangle with sides a, b, c and opposite angles α, β, γ, respectively,

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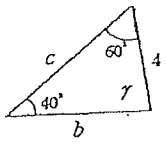
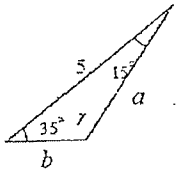
$$\frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}$$

Example 1: Solve the triangle: $\alpha=40^\circ, \beta=60^\circ, a=4$



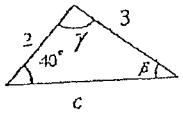
Solution: $\gamma=80^\circ$, $b=\dfrac{4\sin 60^\circ}{\sin 40^\circ}\approx 5.39$, $c=\dfrac{4\sin 80^\circ}{\sin 40^\circ}\approx 6.13$

Example 2: Solve the triangle: $\alpha=35^\circ, \beta=15^\circ, c=5$



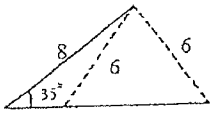
Solution: $\gamma = 130^\circ,\ a = \dfrac{5\sin 35^\circ}{\sin 130^\circ} \approx 3.74,\ b = \dfrac{5\sin 15^\circ}{\sin 130^\circ} \approx 1.69$

Example 3: Solve the triangle: $a = 3, b = 2, \alpha = 40^\circ$

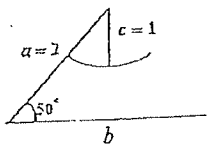


Solution: $\sin\beta \approx 0.43,\ \gamma = 154.6^\circ,\ c = \dfrac{3\sin 114.6^\circ}{\sin 40^\circ} \approx 4.24$

Example 4: Solve the triangle: $a = 6, b = 8, \alpha = 35^\circ$



Solution: $\beta_1 \approx 49.9^\circ$ or $\beta_2 \approx 130.1^\circ$, $\gamma_1 \approx 95.1^\circ$ or $\gamma_2 \approx 14.9^\circ$, $c_1 \approx 10.42$ or $c_2 \approx 2.69$



Example 5: Solve the triangle: $a = 2, c = 1, \gamma = 50^\circ$

Solution: $\sin\alpha = 2\sin 50^\circ \approx 1.53$

1.18 Law of Sines Homework

1. The difference formula for the sine function is $\sin(\alpha - \beta) =$ _____.
2. If θ is a acute angle, solve the equation $\cos\theta = \frac{\sqrt{3}}{2}$
3. A triangle with sides 2 and 5 is similar to a triangle with corresponding sides of 3 and x. Find the missing lengths.
4. If none of the angles of a triangle is a right angle, the triangle is called_____.
5. For a triangle with sides a, b, c and opposite angles α, β, γ, the Law of Sines states that_____.
6. *True of False*: An oblique triangle in which two sides and an angle are given always results in at least one triangle.
7. *True of False*: The sum of the angles of any triangle equals 180°

In problems 17 – 23, solve each triangle.

17. $\alpha = 40^\circ$, $\beta = 20^\circ$, $a = 2$
19. $\beta = 70^\circ$, $\gamma = 10^\circ$, $b = 5$
21. $\alpha = 110^\circ$, $\gamma = 30^\circ$, $c = 3$
23. $\alpha = 40^\circ$, $\beta = 40^\circ$, $c = 2$

In problems 25 – 35, two sides and an angle are given. Determine whether the given information results in one triangle, two triangles, or no triangle at all. Solve any triangle(s) that results.

25. $a=3,\ b=2, \alpha=50^\circ$ 27. $b=5,\ c=3,\ \beta=100^\circ$ 29. $a=4,\ c=6,\ \beta=20^\circ$

31. $b=4,\ c=6,\ \beta=20^\circ$ 33. $a=2,\ c=1,\ \gamma=100^\circ$ 35. $a=2,\ c=1,\ \gamma=25^\circ$

1.19 Law of Cosines

We used the Law of Sines to solve Case 1 (SAA or ASA) and Case 2 (SSA) of an oblique triangle. Now we use the Law of Cosines to solve Case 3 and Case 4.

Case 3: Two sides and the including angle are known (SAS).

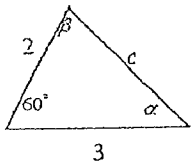
Case 4: Three sides are known (SSS).

Law of Cosines Theorem

For a triangle with sides a, b, c and opposite angles α, β, γ respectively,

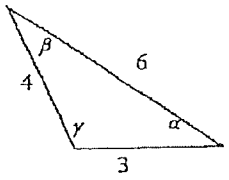
$$c^2 = a^2 + b^2 - 2ab\cos\gamma \quad (1)$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta \quad (2)$$
$$a^2 = b^2 + c^2 - 2bc\cos\alpha \quad (3)$$

Example 1 Solve a SAS Triangle where $a=2,\ b=3,\ \gamma=60^\circ$.



Solution: $c=\sqrt{7}$, $\alpha\approx 40.9^\circ$, $\beta\approx 79.1^\circ$

Example 2 Solve a SSS Triangle where $a=4$, $b=3$, $c=6$.



Solution: $\alpha\approx 36.3^\circ$, $\beta\approx 26.4^\circ$, $\gamma=117.3^\circ$

1.19 Law of Cosines Homework

1. Write the formula for the distance d from $P_1=(x_1,y_1)$ to $P_2=(x_2,y_2)$.

2. If θ is an acute angle, solve the equation $\cos\theta=\frac{\sqrt{2}}{2}$.

3. If three sides of a triangle are given, the Law of _____is used to solve the triangle.

4. If one side and two angles of a triangle are given, the Law of _____is used to solve the triangle.

5. If two sides and the included angle of a triangle are given, the Law of_____ is used to solve the triangle.

6. *True or False*: Given only the three sides of a triangle, there is insufficient information to solve the triangle.

7. *True or False*: Given two sides and the included angle, the first thing to do to solve the triangle is to use the Law of Sines.

8. *True or False*: A special case of the Law of Conies is the Pythagorean Theorem.

In problems 17 – 31, solve each triangle.

17. $a=3$ $b=4$, $\gamma=40^\circ$
19. $b=1$, $c=3$, $\alpha=80^\circ$
21. $a=3$, $c=2$, $\alpha=80^\circ$
23. $a=2$, $b=2$, $\gamma=50^\circ$
25. $a=12$, $b=13$, $c=5$
27. $a=2$, $b=2$, $c=2$
29. $a=5$, $b=8$, $c=9$
31. $a=10$, $b=8$, $c=5$

Exhibit B

[Fwd: [Fwd: Re: Printing of Course Material]]

Subject: [Fwd: [Fwd: Re: Printing of Course Material]]
From: Paul DeRose <paul.derose@pcmail.maricopa.edu>
Date: Thu, 02 Sep 2010 15:29:11 -0700
To: Casandra Kakar <casandra.kakar@pcmail.maricopa.edu>

Subject: Re: Printing of Course Material
From: Ronnie Elliott <ronnie.elliott@pcmail.maricopa.edu>
Date: Tue, 12 Jan 2010 14:14:21 -0700
To: "cleopatria.martinez" <cleopatria.martinez@pcmail.maricopa.edu>

MAT 182 and 187 are the two I'm aware of at this point.

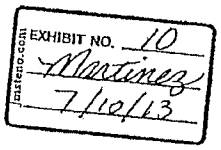
cleopatria.martinez wrote:
> Please remind me, what are the items I had printed for fall that were suspect and the same for my spring requests.
> Ronnie Elliott wrote:
> > Cleopatria,
> >
> > As you may or may not know, we have mechanisms in place to red flag potential copy right issues. Last week it was brought my attention that some of the items you had printed for fall were suspect and the same for your spring requests. Do you have any document of approval from the publisher to use their material? If so, would you please provide that documentation to me so that I can release your requests to be printed. If you do not have such approval, then we will not be able to print your requests at this time.
> >
> > Ronnie

Re: Printing of Course Material.eml	Content-Type: message/rfc822 Content-Encoding: 7bit

EXHIBIT NO. 9
Martinez
7/10/13

MCCCD/Martinez01130

Exhibit C



Subject: Printing of Course Material
From: Ronnie Elliott <ronnie.elliott@pcmail.maricopa.edu>
Date: Tue, 26 Jan 2010 07:52:16 -0700
To: Cleopatria Martinez <cleopatria.martinez@pcmail.maricopa.edu>
CC: Ronnie Elliott <ronnie.elliott@pcmail.maricopa.edu>, Lee Combs <lee.combs@domail.maricopa.edu>, Margaret McConnell <margaret.mcconnell@domail.maricopa.edu>, Anna Solley <anna.solley@pcmail.maricopa.edu>, Casandra Kakar <casandra.kakar@pcmail.maricopa.edu>

Cleopatria,

In order to respond to your email dated January 20, 2010, I solicited the guidance of our district legal counsel, Margaret McConnell, regarding copyright infringement and fair use as defined in federal law. I did provide Margaret with copies of your requests and a copy of the adopted text book, and this morning received her findings based on the federal copyright law, including the limitations of fair use.

The District's Administrative Regulation 3.2 provides in part as follows:

4. Employees are prohibited from copying materials not specifically allowed by the (1) copyright law, (2) fair use guidelines, (3) licenses or contractual agreements, or (4) other permission.

5. The Governing Board disapproves of unauthorized duplication in any form. Employees who willfully disregard this Board policy and/or the aforementioned copyright guidelines do so at their own risk and assume all liability for their actions.

The focus here is whether your use of certain materials meets one of the criteria specified in Para. 4 quoted above. We don't have any information that suggests that your use fits within the specific exception under the copyright law authorizing the use of copyrighted materials without permission. Nor have we seen any licenses or written permission from the copyright holder, Pearson, of the materials for use of their materials in your lecture notes. Therefore, we must focus on "fair use."

The doctrine of fair use allows for the use of copyrighted works without the owner's permission. It protects limited uses of copyrighted works from being an infringement. The doctrine is, however, determined on a case-by-case basis.

The federal Copyright Act provides that the "fair use of a copyrighted work for purposes such as criticism, comment, news reporting, teaching (including multiple copies for classroom use), scholarship, or research, is not an infringement of copyright." Again, not all educational uses are fair use. Four factors are considered in determining whether the use of a work is a fair use:

1. the purpose and character of the use, including whether such use is of a commercial nature or is for nonprofit educational purposes;
2. the nature of the copyrighted work;
3. the amount and substantiality of the portion used in relation to the copyrighted work as a whole; and
4. the effect of the use upon the potential market for or value of the copyrighted work.

17 U.S.C. § 107. Courts decide fair use by weighing each factor individually and then determining which way the combination of factors tips.

In light of those legal standards, we have reviewed your January 20 e-mail and the following materials - a document entitled "Phoenix College Fall 2009, MAT 182 Trigonometry Section, Lecture Notes, Dr. Cleopatria Martinez" and a book, Precalculus: Concepts Through Functions: A Right Triangle Approach to Trigonometry, Instructor's Edition, authored by Michael Sullivan and Michael Sullivan, III. The book indicates that the copyright holder is Pearson Education, Inc. and it says: "All rights reserved. No part of this book may be reproduced, in any form or by any means, without permission in writing from the publisher."

The first issue here is whether your lecture notes are original works or problems created by you, or whether portions are instead copied from the Precalculus book or some other copyrighted source

In reviewing parts of your notes and the book, we have some real concerns. Attached are some pages of your notes on which we have

handwritten the page numbers of the Prec. as
book as the source of the example, text or problem. Once we came up with these 10 or so correlations, we stopped looking, as the problem seemed clear. The examples or problems are copied verbatim into the lecture notes.

Additionally, there is no attribution in each of these instances of the source of the example or problem, that is, no identification of Pearson as the copyright holder. Whether you intend to or not, the lecture notes give the distinct impression that they are your original work, which is incorrect. That lack of attribution is, by itself, a legal issue. As to what the breadth of the problem is, only you know that because we don't know all of the resources that you may have used for those notes.

Moreover, it is our understanding that you may have been using these notes over a period of time.

As you can see from the listing of the "fair use" requirements, the use must not be one that, if it became widespread over an entire market, would reduce the potential market for the copyrighted work. In this case, copying problem or examples from the book, with the potential that the students don't need to purchase it, would make your use without permission a problem.

Other guidelines developed under the auspices of the federal government specify that use must "spontaneous." It is hard to make a case that this is spontaneous use if you use the materials over and over each semester. In any event, it would appear to have taken a significant effort to copy the problems and examples into the text of your lecture notes, thereby eliminating any "spontaneity."

Based on the above, we believe that you need to get written permission to use portions of the book in your lecture notes from Pearson, and make sure that you note that Pearson is the copyright holder of materials. Publishers such as Pearson have specific staff dedicated to this issue. You should find out who those are, identify the portions of the book that you wish to use, and obtain permission in writing.

Bottom line, Cleopatria, you must secure written permission from the publisher before we are able to print your material.

Regards
Ronnie

Lecture Notes and Book.pdf Content-Type: application/pdf
Content-Encoding: base64

EXHIBIT 6

UNITED STATES DISTRICT COURT

DISTRICT OF ARIZONA

Cleopatria Martinez, Plaintiff, v. Maricopa County Community College District, Defendant.	)	CV12-00702-PHX-DGC

VIDEOTAPED DEPOSITION OF CLEOPATRIA MARTINEZ, Ph.D.

(VOLUME 1, Pages 1 through 187)

Phoenix, Arizona
July 10, 2013
9:08 a.m.

Prepared by:
Janet Hauck, RPR
Arizona Certified
Reporter Number 50522

A. Mm-hmm.

Q. -- would you clip the documents together? Would you put them in a file folder? How would you store these documents?

MR. STROJNIK: Object to the form of the question.

Go ahead.

THE WITNESS: In whatever form they were given to me, I would keep them.

Q. BY MS. BALCH: Do you recall ever having applicant information in manila envelope -- in manila folders?

A. Yes.

Q. Do you recall whether manila folders had the applicant names on them?

A. I don't recall.

Q. Please take a look at paragraph 15 of your complaint. States, "Defendants have accused, criticized, and reprimanded Professor Martinez regarding the content of her lecture notes." What are these lecture notes that you're referring to? Can you please describe them for me?

A. They are the guide that I used for teaching.

Q. What do they typically look like?

A. They describe whatever the topic is that I'm

teaching, and they develop the concept and provide examples.

Q. Are they typically bound, stapled, clipped?

A. Do they come -- I present them both ways.

Q. Do they contain a cover page?

A. Some do.

Q. They contain table of contents?

A. Yes, some do.

Q. Page numbers?

A. Yes.

Q. How many pages are these lecture notes, typically?

A. They vary. I could have just a couple of lecture notes, or I can have up to -- pages or up to 100 or 150 pages. Some -- they vary.

Q. Please explain all -- strike that.

Can you please explain all facts that you have in support of your contention that MCCCD has criticized and reprimanded you regarding the content of your lecture notes as referenced in paragraph 15 of your complaint?

MR. STROJNIK: Form.

THE WITNESS: Paragraph 15?

Q. BY MS. BALCH: Correct.

A. I -- I've already given you some. Actually

Q. -- in a week?
A. Yes.
Q. Did you meet all of these students?
A. Yes.
Q. What did you have these students do?
A. They made copies for me. They typed material up for me. They did research. They stapled. They collated. They transcribed what I had written. They ran errands when I needed something taken somewhere to another colleague or a department.
Q. Anything else?
A. Not that I can think of.
Q. Did they do any filing for you, any data entry for you?
A. No.
Q. You mentioned that they typed materials for you. What materials did they type?
A. My notes. I'd write notes, I -- my lecture notes, or tests when I'd write them.
Q. What do you mean they would type notes? How would they type the notes?
A. On a computer, they would type.
Q. But would you -- what would you give the students so that they could type up your notes?
A. A written document that they would type up for

me.

Q. Describe that written document. Was it a piece of notebook paper --

A. Yes --

Q. -- that you had written on?

A. -- that I had written on.

Q. And what were these materials that they were typing?

A. They could be tests, quizzes, handouts, my own lecture notes, lists of formulas, any handout that I wanted them to have -- the students to have. That's -- that's it. My syllabi, sometimes I'd have them type up my syllabi.

Q. Did you ever provide your work study students with a textbook and instruct them to --

A. Yes.

Q. Okay.

A. To -- I'd say in my notes I'd refer to the problem in the textbook that I wanted inserted at a point.

Q. Do you remember what textbooks you utilized in this manner?

A. Just a Sullivan & Sullivan.

Q. And the materials that these work study students were working on, what courses were those for?

MR. STROJNIK: Objection as to form.

Are we still in fall 2009, or are we discussing all work study students that she's always had? I remember you began with fall 2009.

Q. BY MS. BALCH: Mm-hmm. Yeah, we were in fall 2009, so in fall 2009, what --

A. Oh, okay, okay. So what was the question?

Q. In fall of 2009 what -- strike that.

In fall of 2009 were you having your work study students type lecture notes and class materials for you?

A. Yes.

Q. What classes were these lecture notes and course materials for?

A. Whatever I was teaching fall of 2009, tests, quizzes, and handouts.

Q. Did they type answer keys for you?

A. No, I don't think so.

Q. Did these work study students perform any additional tasks?

A. They made -- I've already named all the tasks.

Q. Other than what you've told me so far, did these work study students perform any other tasks for you?

A. Not that I recall.

MR. STROJNIK: From --

THE WITNESS: Go ahead.

MR. STROJNIK: From the spring of '09 --

THE WITNESS: Okay.

MR. STROJNIK: -- through the present, other than this document, which is Exhibit 3, marked MAT 082, Spring 2010 -- other than this document, did you use any other -- in any other courses, did you solely use your lecture notes as the textbook?

THE WITNESS: No, I did not. Just this one.

Q. BY MS. BALCH: Prior to January of 2009, had you ever used your lecture notes solely as the textbook for a math course?

A. No.

Q. How did you go about creating these lecture notes?

MR. STROJNIK: Object to the form of the question.

Q. BY MS. BALCH: Do you understand my question?

A. Which lecture notes are you asking about?

Q. For example, let's take MAT 082 --

A. Okay.

Q. -- the one labeled, Chapters for Basic Arithmetic. It's a voluminous document --

A. Mm-hmm.
Q. -- right?
A. Mm-hmm.
Q. It's --
A. Yes.
Q. -- comprised of -- looks like 105 pages. Does that look about right to you?
A. Yes.
Q. I'm assuming this took quite a bit of time to prepare?
A. Yes.
Q. How did you prepare this document?
A. Well, over the years I've written this information many times, and so some of it was already written that I had written in previous years, my previous 40 years, and some I sat down and I wrote them. I organized them, and I -- I had material from many, many different people that I used to put this together.
Q. Did you type this document?
A. Yes, I did.
Q. The entire document, or did you have assistance in typing this document?
A. I believe I typed this one. I may have had my work study students -- I may have given them pages that were already written, and they typed it. They typed

those pages, but I don't remember which pages those are.

Q. So let me make sure I understand this correctly. Your testimony is that either you typed this document --

A. Mm-hmm.

Q. -- or in instances where you might have had work study students type portions of this for you, you would have, what, written it out on a scratch piece of paper and handed it to the work study students for them to type?

A. Yes, that's right.

Q. Did you provide any textbooks to your work study students to copy problems from to insert into the lecture notes at Exhibit 3, titled, Chapter for Basic Arithmetic?

MR. STROJNIK: Object to the form of the question.

You can answer if you can.

THE WITNESS: No textbooks were used.

Q. BY MS. BALCH: No textbooks were used?

A. I credited the people from whom I had documents on the thank you page.

Q. But on the credit page there are no publishers of textbooks, correct?

A. Correct.

Q. And is it your contention that you or work study students acting on your behalf did not copy any math problems from a textbook to be inserted into these lecture materials?

MR. STROJNIK: Object to the form of the question.

THE WITNESS: State it again, please.

MR. STROJNIK: Can you add "to your knowledge" to that question?

MS. BALCH: Certainly.

Q. BY MS. BALCH: Actually, I'll ask -- I'll ask it in two separate parts.

A. Okay.

Q. That should make things a little bit easier.

When you typed -- back up. So this document was created in one of two manners. Either you typed the document and/or you assigned to your work study students certain pages that you had handwritten and told them to type up, correct?

A. Yes.

Q. So for the portions that you typed personally --

A. Mm-hmm.

Q. -- did you copy, or borrow, or verbatim utilize any math problems that you found in math

textbooks?

A. No, absolutely not.

Q. As far as the portions that you gave to your federal work study students to type, in your handwritten notes that you provided to the work study students, did you indicate to your federal work study students that you wanted them to copy certain problems from certain textbooks to incorporate into the lecture notes they were typing?

MR. STROJNIK: Object to the form of the question.

You can answer if you're able to.

THE WITNESS: No.

Q. BY MS. BALCH: You understand my question? I know it's getting late in the day, but do you understand my question?

A. Say it again, please.

Q. Earlier today you testified that when work study students would type your lecture notes for you, sometimes you would put a note in your handwritten sheets where you would ask the students to insert math problems from a certain textbook; do you recall us discussing that earlier today?

A. Yes.

Q. When you were preparing the lecture notes for

Math 082 that are at Exhibit 3, when you were handwriting out the pages that you wanted your federal work study students to type, did you instruct them to copy math problems from textbooks?

A. No, absolutely not.

Q. Do you still have those pages --

A. No.

Q. -- of handwritten notes?

A. Oh, no, that was four or five years ago. No.

Q. When was the last time that you created lecture notes of the type that are found in Exhibit Number 3?

MR. STROJNIK: Object to the form of the question.

THE WITNESS: I'm not sure if you're asking me for a bundle of them, or one page of them, or -- what do you consider lecture notes?

Q. BY MS. BALCH: I -- I'm using your term, lecture notes. And when I've asked you to describe what your lecture notes are, I think we can agree that you'd characterize these three packets at Exhibit Number 3 to be lecture notes, correct?

MR. STROJNIK: I believe she testified these were textbooks and lecture notes.

THE WITNESS: Right.

the students do not have to buy the textbook?

MR. STROJNIK: Object to the form of the question.

Go ahead.

THE WITNESS: No, I -- I just teach -- I teach the content of the subject, and I select what I think is best in terms of content and what's relative and understandable, best understood by my students, what is the best product for my class. That's how I select a textbook.

Q. BY MS. BALCH: Do you always assign a textbook in your courses?

A. Yes. Yes, I do.

MR. STROJNIK: Form.

THE WITNESS: I do.

MS. BALCH: Just want to make sure we have a clear record of that.

THE REPORTER: Can we go off the record for a second?

THE VIDEOGRAPHER: Off the record at 1:24 p.m.

(Brief recess.)

THE VIDEOGRAPHER: On the record at 1:25 p.m.

Q. BY MS. BALCH: Dr. Martinez, I just want to

make sure that we have a clear record here. There was a little bit of confusion on the record, and I think that some of us spoke over each other, so I'm going to ask this question again.

Do you always assign a textbook in all of the courses -- in all of the math courses that you teach at Phoenix Community College?

MR. STROJNIK: Object to the form of the question.

THE WITNESS: I assign what I call a textbook in all my courses.

Q. BY MS. BALCH: And what do you call a textbook?

A. A series of pages that has the content that I'm teaching.

Q. Are some of these textbooks your lecture notes?

A. They have been.

Q. For what classes?

A. Are we -- in which time frame?

Q. Let's talk from January of 2009 to the present.

A. January -- and what are you asking me again about that time frame?

Q. Classes where the assigned -- where the

assigned textbook is your lecture notes.

A. In January of 2010?

Q. January of 2009 through the present.

MR. STROJNIK: She's asking starting January 2009 for which of your classes did you use your lecture notes as the textbook?

THE WITNESS: Oh, okay. It was for a -- I believe it was a college algebra class.

Q. BY MS. BALCH: Do you recall the course number for that college algebra course?

A. No, I don't.

Q. Any other courses?

A. From January of 2009 forward -- January?

Q. Yes.

A. I may have used my notes for an arithmetic class, Math 108.

Q. 108?

A. Mm-hmm -- oh, no wait, I'm sorry, not 108. Math 082, 082.

Q. Any other courses?

MR. STROJNIK: Do you have any -- I'm sorry, do you have any documents that could possibly refresh her recollection? Because I know there's a ton of documents that we can look at that would help her.

MS. BALCH: I -- I -- I have some

documents, but for courses that she's -- well --

MR. STROJNIK: Or if you have them by memory, maybe you can ask them specifically.

MS. BALCH: Sorry, Counsel, I don't have that in my fingertips.

MR. STROJNIK: Okay. Do you have that in your memory bank, because you could just ask her -- I mean, it's at center stage pretty much, but go ahead, to the best of your recollection.

THE WITNESS: I don't -- you know -- I don't remember what I was teaching. That was four or five years ago. I don't remember, but I believe I taught the two classes I mentioned using my lecture notes.

MR. STROJNIK: She's asking from January 2009, fall, spring -- fall, 2009, spring, fall, 2010 and on and on --

THE WITNESS: Right, right.

MR. STROJNIK: -- if you can --

THE WITNESS: Well, I -- I was precluded from using notes after spring 2010. I haven't been able to use notes since then.

(Exhibit No. 3 was marked.)

Q. BY MS. BALCH: You've been handed what's been marked as Exhibit 3. Exhibit 3 consists of three packets of documentation.

A. Mm-hmm.
Q. One -- one is labeled Chapters for Basic Arithmetic?
A. Mm-hmm.
MR. STROJNIK: Is that a yes?
THE WITNESS: Yes.
Q. BY MS. BALCH: The next is marked at the top, MAT 182 Trigonometry, Spring of 2010; do you see that?
A. Yes.
Q. And the final is labeled, MAT 182 Precalculus, Trigonometry Section Lecture Notes for fall of 2009; is that correct?
A. Yes.
Q. Do you recognize these documents?
A. Yes.
Q. What are these documents?
A. They are my lecture notes for the subjects.
Q. And do these packets of lecture notes appear to be true and accurate representations of the lecture notes that you utilized in these courses?
MR. STROJNIK: I'm going to -- in light of the allegations in this lawsuit, I would recommend that you thoroughly review these exhibits to ensure they are accurate.
MS. BALCH: Take your time.

MR. STROJNIK: Yes, take your time. This is important.

If you'd like, so we don't use up your time, do you want to do this off the record so we don't use your seven hours? Because I am going to recommend that -- there's copyright allegations in this lawsuit. We have to make sure that each page is actually the actual page and something isn't inserted in there unbeknownst to us. I'm not saying intentionally, but maybe mistakenly by someone.

MS. BALCH: Absolutely, Counsel. How about this? Can we agree to stipulate or to -- to basically stipulate to whether or not these are true or accurate representations after you've had the opportunity to review them?

MR. STROJNIK: Sure, okay.

MS. BALCH: And that's Exhibit 3, all three of these packets, so --

MR. STROJNIK: So we're reserving the right later to say that these are not accurate.

THE WITNESS: Okay.

Q. BY MS. BALCH: And with that caveat, these generally appear to be the types of lecture notes that we have been discussing; is that correct?

A. Yes.

Q. And these are the types -- and at least for two of the courses that you have taught since January of 2009, these are the types of materials that you used as a textbook?

A. Yes.

Q. And one of those courses was Math 082?

A. Yes.

Q. And this first document in this packet, it's Bates numbered MCCCD/Martinez 00528 through 636. If you flip to the third page of that packet, do you see at the top it says, MA2 -- MAT 082, Spring 2010 outline?

A. Yes.

Q. While -- subject, of course, to you and your counsel's review of this document on a page-by-page basis to ensure that there are no inadvertent documents that have been omitted or inserted, does this packet appear to be the text that you utilized for Math 082?

A. Yes.

MR. STROJNIK: Object to form.

THE WITNESS: Yes.

Q. BY MS. BALCH: And was this the only assigned textbook that you utilized for your Math 082 course in spring of 2010?

A. I did use this in 2010? See, I'm not sure what I taught in 2010.

Q. Well, I'm -- I'm asking you, Dr. Martinez, and at the top of the third page --
A. Mm-hmm.
Q. -- of that packet --
A. Oh, okay.
Q. -- it says --
A. Oh, I see.
Q. Or wait, one, two, three. It says MAT 082, Spring 2010 Outline?
A. Yes.
Q. So does this document appear to be the textbook that you utilized in spring of 2010 for your MAT 082 course?
A. Yes.
Q. And other than this packet, did you utilize any other assigned textbooks for your MAT 082 spring 2010 course?
A. No.
Q. Let's take the next packet. At the top it says -- I'm looking at the one that says, MAT 182 Trigonometry.
A. Mm-hmm, yes.
Q. And the Bates range is MCCCD/Martinez 00400 through 00450.
A. Yes.

MR. STROJNIK: You know what Bates is, right? Okay.

Q. BY MS. BALCH: Does this appear to be the lecture notes that you utilized in your spring of 2010 MAT 182 Trigonometry course?

A. Yes.

Q. And was this the only assigned textbook in that course?

A. No.

Q. What other textbook was assigned?

A. I don't remember, but there was another textbook.

Q. How many assigned textbooks?

A. One.

Q. Do you recall either the name or the author of the textbook?

A. No, I don't.

MR. STROJNIK: Oh, boy, looking at this stuff, I'm glad I'm through with all of this.

Q. BY MS. BALCH: Last packet is labeled, MAT 182 Precalculus, Fall 2009.

A. Mm-hmm, yeah, yes.

Q. And the Bates number at the bottom is MCCCD/Martinez 00451 through 526.

A. Yes.

Q. Do -- does this packet appear to be the textbook that you utilized in your MAT 182 course in fall of 2009?

A. Yes.

Q. Did you require any other assigned textbooks for that course?

A. I do not recall.

Q. And earlier you testified that you believed you used lecture notes similar to those contained in Exhibit 3 as the textbook for at least two courses -- two math courses from January 2009 to the present. One was Math 082. The other one was College Algebra. Is that correct?

A. Okay, now that I've got these documents --

Q. Mm-hmm.

A. -- I wasn't sure about that. Would you go through the question again?

Q. Absolutely. From January 2009 through the present, what courses did you teach where you used your lecture notes as the textbook for the course?

A. As the textbook?

MR. STROJNIK: Object to the form of the question.

Q. BY MS. BALCH: Let me clarify. What -- for the classes that you taught from January 2009 to the

present, please identify for me the courses that you taught where the -- where your lecture notes, similar to those in Exhibit 3, were the only textbook --

A. Oh, okay.

Q. -- that you utilized for the course?

A. That would be the Math 082, spring 2010.

Q. Are there any other courses --

A. No.

Q. -- where -- let me -- let me finish the --

A. Oh, okay.

Q. -- the question first.

Are there any other courses from January 2009 through the present where the -- your lecture notes were the only assigned textbook for the course other than MAT 082?

A. And the timeline was?

Q. January of 2009 through the present.

A. Where my lecture notes, is that what you said?

Q. Your lecture notes, similar to those in Exhibit 3 to your deposition.

A. Okay. This is the only one, the Math 082.

MR. STROJNIK: Do you understand the question?

THE WITNESS: I'm -- I think I do. Shall I say it, because I'm not sure. I'm not sure.

MR. STROJNIK: From --
THE WITNESS: Go ahead.
MR. STROJNIK: From the spring of '09 --
THE WITNESS: Okay.
MR. STROJNIK: -- through the present, other than this document, which is Exhibit 3, marked MAT 082, Spring 2010 -- other than this document, did you use any other -- in any other courses, did you solely use your lecture notes as the textbook?
THE WITNESS: No, I did not. Just this one.
Q. BY MS. BALCH: Prior to January of 2009, had you ever used your lecture notes solely as the textbook for a math course?
A. No.
Q. How did you go about creating these lecture notes?
MR. STROJNIK: Object to the form of the question.
Q. BY MS. BALCH: Do you understand my question?
A. Which lecture notes are you asking about?
Q. For example, let's take MAT 082 --
A. Okay.
Q. -- the one labeled, Chapters for Basic Arithmetic. It's a voluminous document --

A. Mm-hmm.
Q. -- right?
A. Mm-hmm.
Q. It's --
A. Yes.
Q. -- comprised of -- looks like 105 pages. Does that look about right to you?
A. Yes.
Q. I'm assuming this took quite a bit of time to prepare?
A. Yes.
Q. How did you prepare this document?
A. Well, over the years I've written this information many times, and so some of it was already written that I had written in previous years, my previous 40 years, and some I sat down and I wrote them. I organized them, and I -- I had material from many, many different people that I used to put this together.
Q. Did you type this document?
A. Yes, I did.
Q. The entire document, or did you have assistance in typing this document?
A. I believe I typed this one. I may have had my work study students -- I may have given them pages that were already written, and they typed it. They typed

those pages, but I don't remember which pages those are.

Q. So let me make sure I understand this correctly. Your testimony is that either you typed this document --

A. Mm-hmm.

Q. -- or in instances where you might have had work study students type portions of this for you, you would have, what, written it out on a scratch piece of paper and handed it to the work study students for them to type?

A. Yes, that's right.

Q. Did you provide any textbooks to your work study students to copy problems from to insert into the lecture notes at Exhibit 3, titled, Chapter for Basic Arithmetic?

MR. STROJNIK: Object to the form of the question.

You can answer if you can.

THE WITNESS: No textbooks were used.

Q. BY MS. BALCH: No textbooks were used?

A. I credited the people from whom I had documents on the thank you page.

Q. But on the credit page there are no publishers of textbooks, correct?

A. Correct.

Q. And is it your contention that you or work study students acting on your behalf did not copy any math problems from a textbook to be inserted into these lecture materials?

MR. STROJNIK: Object to the form of the question.

THE WITNESS: State it again, please.

MR. STROJNIK: Can you add "to your knowledge" to that question?

MS. BALCH: Certainly.

Q. BY MS. BALCH: Actually, I'll ask -- I'll ask it in two separate parts.

A. Okay.

Q. That should make things a little bit easier.

When you typed -- back up. So this document was created in one of two manners. Either you typed the document and/or you assigned to your work study students certain pages that you had handwritten and told them to type up, correct?

A. Yes.

Q. So for the portions that you typed personally --

A. Mm-hmm.

Q. -- did you copy, or borrow, or verbatim utilize any math problems that you found in math

textbooks?

A. No, absolutely not.

Q. As far as the portions that you gave to your federal work study students to type, in your handwritten notes that you provided to the work study students, did you indicate to your federal work study students that you wanted them to copy certain problems from certain textbooks to incorporate into the lecture notes they were typing?

MR. STROJNIK: Object to the form of the question.

You can answer if you're able to.

THE WITNESS: No.

Q. BY MS. BALCH: You understand my question? I know it's getting late in the day, but do you understand my question?

A. Say it again, please.

Q. Earlier today you testified that when work study students would type your lecture notes for you, sometimes you would put a note in your handwritten sheets where you would ask the students to insert math problems from a certain textbook; do you recall us discussing that earlier today?

A. Yes.

Q. When you were preparing the lecture notes for

Math 082 that are at Exhibit 3, when you were handwriting out the pages that you wanted your federal work study students to type, did you instruct them to copy math problems from textbooks?

A. No, absolutely not.

Q. Do you still have those pages --

A. No.

Q. -- of handwritten notes?

A. Oh, no, that was four or five years ago. No.

Q. When was the last time that you created lecture notes of the type that are found in Exhibit Number 3?

MR. STROJNIK: Object to the form of the question.

THE WITNESS: I'm not sure if you're asking me for a bundle of them, or one page of them, or -- what do you consider lecture notes?

Q. BY MS. BALCH: I -- I'm using your term, lecture notes. And when I've asked you to describe what your lecture notes are, I think we can agree that you'd characterize these three packets at Exhibit Number 3 to be lecture notes, correct?

MR. STROJNIK: I believe she testified these were textbooks and lecture notes.

THE WITNESS: Right.

Q. BY MS. BALCH: Okay. What -- what would you like to refer to the documents at Exhibit Number 3 as? I'll use whatever term you --

A. This is the -- the textbook.

Q. This is the textbook, okay. Have you created -- since spring of 2010 --

A. Mm-hmm.

Q. -- have you created any new textbooks of the type that are -- that are at Exhibit 3?

A. No.

Q. Since spring of 2009, what courses have you created textbooks for?

A. This is the only one, the 082.

Q. So in courses where you create these types of materials --

A. Mm-hmm.

Q. -- but you also have a separate assigned textbook by a publisher, do you still refer to these notes as your textbook?

MR. STROJNIK: Object to the form of the question.

THE WITNESS: These notes meaning the 082 notes or --

Q. BY MS. BALCH: In general, so these notes --

A. Okay.

Q. -- at the -- the other two packets at Exhibit 3.

A. I do not call them textbooks. They are lecture notes.

Q. So the distinction is, if I understand this correctly, if -- with respect to the documents that you prepare to be utilized as course materials --

A. Mm-hmm, yes.

Q. -- if it is the only course materials, you refer to it as a textbook?

A. Yes.

Q. If it is designed to be used in addition to a textbook from a publisher, you refer to them as lecture notes?

A. Yes.

Q. Why did you create your own textbook for MAT 082?

MR. STROJNIK: Object to the form of the question.

THE WITNESS: Because -- I -- I created it, because I knew what the content was that students needed to learn, and I -- I know how I present it. I think I present it better in a more clear fashion. This is more customized to the students at Phoenix College in -- in vocabulary, and presentation, and in order,

and -- and in breadth, in depth, whereas textbooks are -- don't -- I didn't think fit the students as well. So I created my own textbook.

Q. BY MS. BALCH: Did you create the textbooks in order to help students save money?

A. I -- I -- I just answered why I created them, because of the content and the presentation, the quality of the questions, the presentation, the order, the development of the concept. I have many years of experience that has shown me some techniques and problems that help students -- that motivate students and help them learn best, and I wanted a textbook that reflected the best.

Q. So is it your testimony that trying to help students save money did not have any bearing --

MR. STROJNIK: Asked and answered. Come on.

Q. BY MS. BALCH: -- on your decision to create these lecture notes?

MR. STROJNIK: Object to the form of the question.

Don't answer that. You've answered this three, four times already.

Move on to the next line of questioning.

Q. BY MS. BALCH: Dr. Martinez, let's talk about

UNITED STATES DISTRICT COURT

DISTRICT OF ARIZONA

Cleopatria Martinez, Plaintiff, v. Maricopa County Community College District, Defendant.	)	CV12-00702-PHX-DGC

VIDEOTAPED DEPOSITION OF CLEOPATRIA MARTINEZ, Ph.D.

VOLUME 2 (Pages 188 through 278)

Phoenix, Arizona
July 26, 2013
11:12 a.m.

Prepared by:
Janet Hauck, RPR
Arizona Certified
Reporter Number 50522

can see the date that was on there, but my best -- I would think -- I think it was after these.

MS. BALCH: Thank you.

All right. Do you want to take that break real quick, Counsel --

MR. STROJNIK: Sure.

MS. BALCH: -- confer with your client?

THE VIDEOGRAPHER: Off the record at 12:02.

(Recess from 12:02 p.m. to 12:08 p.m.)

THE VIDEOGRAPHER: On the record at 12:09.

Q. BY MS. BALCH: Dr. Martinez, do you know who -- I might mispronounce his name, but Johnny Santellan is?

A. Yes, I do.

Q. Who is he?

A. He's an adjunct math instructor.

Q. Would you consider him to be a friend?

A. Yes, I would, a friend and a colleague.

Q. Do you know what race he is?

A. No.

Q. Do you know what national origin he is?

A. I believe he's Mexican.

Q. Have you ever asked him to make photocopies for you?

A. Yes.

Q. When?

A. I don't recall what the date was.
Q. What did you ask him to make photocopies of?
A. Over a weekend I had written up a page of inverse trig function graphs because the book didn't have them all together, and it didn't show them all, and I wanted my students to know what they looked like and have information about them. So I spent the entire weekend writing this one page up and putting the information the way I thought -- the best way I thought it would be, and Monday morning I -- but I did it over the weekend, and Monday morning I wanted a copy, but I couldn't find Joe Sueyoshi, and I couldn't find Casandra Kakar, and I wanted to use it in my class.

And I knew it wasn't a copyright violation, because I had written it myself that weekend, so I thought since it's not a copyright violation because I wrote it, I -- I know that, because I just did it all weekend long it took me. I asked Johnny if he would make a copy, and I would -- I was going to hold them until Joe said okay, because it was so close to classroom time, and if he didn't say okay, then I wasn't going to use them.
Q. Did you ever get the photocopies back from Mr. -- is it Santellan?
A. Santellan.
Q. Santellan. Did you ever get the photocopies

Q. Did you ever ask him to use his course materials for one or more of your classes that you were going to be teaching in fall of 2012?
A. Several times.
Q. What did he say?
A. In 2012?
Q. Yes.
A. Ask the question again, please.
Q. Did you ever ask him to use his course materials for any classes that you were teaching in the fall of 2012?
A. Yes.
Q. What classes and what did he say?
A. Right now, I don't remember. The class was -- he gave me permission. He said he -- spring -- no, I don't remember what I taught spring or what I taught fall this year, so I can't remember. I'm getting the two mixed up.
Q. Did you use those materials for the courses that you were teaching?
A. Yes, I used it as a textbook.
Q. Did you distribute materials to students?
A. I did.
Q. How did you distribute them to students?
A. Well, at the beginning of the semester --

because this was a last minute change -- I asked the students whether they wanted to make their own copies, and I needed to have them done quickly, because this was already the first or second day of classes, or whether they wanted me to make the copies and then they would reimburse me for whatever it cost me to make the copies. And that way, we would all have them at the same time. And all but one student said they wanted me to make them, and then the next class period, the one remaining -- one student said, no, you make mine, too, please.

So I went to Staples, and I had copies made and bound. I came back and I took them to class, and I told the students at first it would be 20 -- about $20. I did not know what it was going to cost. And it turned out to cost a little bit over $11, and I told them $11. And so I told them that they need to reimburse me for the -- and they agreed, so we exchanged the book for the $11.

Q. And did the students pay that money directly to you?

A. Yes.

Q. Did you submit -- strike that.

MR. STROJNIK: That's a good question actually -- ask.

Q. BY MS. BALCH: This copy job that you processed

change or not, because the grade is still A.

Q. Dr. Martinez, you used to be the math department chair; is that correct?

A. Yes.

Q. What were your job duties and responsibilities as a math department chair?

A. I represented the math department faculty interests to the administration, and I communicated messages from the administration to the math -- to the math department. I scheduled -- I wrote -- wrote the schedule each semester. I -- I dealt with any complaints from students or among faculty. I supervised the chair -- the -- the secretary, and all the workers, work study, the students, et cetera. I made sure the department ran smoothly and that the needs of the faculty were heard by the administration.

Q. Did you have the ability to hire anyone?

A. No, the chair does not hire.

Q. Did you have the ability to fire anyone?

A. No, the chair does not have -- fire.

Q. Did you have the ability to demote anyone?

A. No.

Q. Or reduce anyone's amount of pay?

A. The chair does not have that authority.

Q. Was it -- would you characterize it as more of

an administrative position?

MR. STROJNIK: Object to the form of the question.

Answer if you're able.

THE WITNESS: Faculty don't necessarily consider that administrative, although that might be changing, so I can't answer it right now.

Q. BY MS. BALCH: Dr. Martinez, are you claiming that there was some sort of policy or widespread practice that existed that caused you harm?

MR. STROJNIK: Object to the form of the question.

Answer if you are able.

THE WITNESS: Would you restate that question?

Q. BY MS. BALCH: Yeah. Are you claiming that there was a policy or a practice that existed that was discriminatory?

A. A policy?

MR. STROJNIK: Object to the form of the question.

Answer if you're able to.

THE WITNESS: I can't address the -- I don't know about policy.

Q. BY MS. BALCH: Do you know what a policy is?

EXHIBIT 7

Comparison Between Charles P. McKeague "Basic Mathematics" Textbook & Plaintiff's MAT082 Lecture Notes

Basic Mathematics Textbook. By: Charles P. McKeague. p. 285

4.1 Problem Set 225

31. Family Budget A family of four budgeted the following amounts for some of their monthly bills.

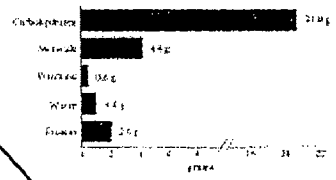
a. What is the ratio of the rent to the food bill?

b. What is the ratio of the gas bill to the food bill?

c. What is the ratio of the utilities bill to the food bill?

d. What is the ratio of the rent to the utilities bill?

32. Nutrition One cup of breakfast cereal was found to contain the following nutrients.



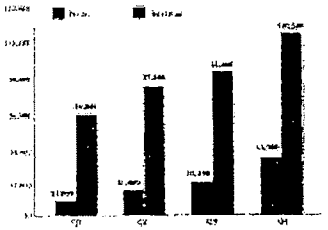
a. Find the ratio of water to protein.

b. Find the ratio of carbohydrates to protein.

c. Find the ratio of vitamins to minerals.

d. Find the ratio of protein to vitamins and minerals.

33. Profit and Revenue The following bar chart shows the profit and revenue of the Baby Steps Shoe Company for each quarter for one year.

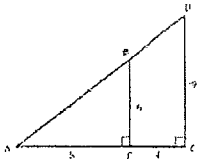


Find the ratio of revenue to profit for each of the following quarters. Write your answer in lowest terms.

a. Q1 b. Q2 c. Q3 d. Q4

e. Find the ratio of revenue to profit for the entire year.

34. Geometry Regarding the diagram below, AC represents the length of the line segment that starts at A and ends at C. From the diagram we see that $AC = 8$.



a. Find the ratio of BC to AC.

b. What is the length AE?

c. Find the ratio of DE to AE.

MAT082 Lecture Notes. Prepared by Plaintiff. p. 49

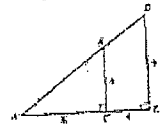
2.9 Ratios Homework

Give the ratio of the following quantities.

1. $5 to 6	2. 6 to 8	3. 64 to 12	4. 12 to 64
5. 100 to 250	6. 250 to 100	7. 13 to 26	8. 36 to 18
9. $\frac{3}{4}$ to $\frac{1}{4}$	10. $\frac{5}{8}$ to $\frac{3}{8}$	11. $\frac{7}{3}$ to $\frac{6}{3}$	12. $\frac{9}{5}$ to $\frac{11}{5}$
13. $\frac{6}{5}$ to $\frac{6}{7}$	14. $\frac{5}{3}$ to $\frac{1}{3}$	15. $2\frac{1}{2}$ to $\frac{1}{2}$	16. $5\frac{1}{4}$ to $\frac{3}{4}$
17. $2\frac{2}{3}$ to $\frac{5}{3}$	18. $\frac{1}{2}$ to $\frac{1}{2}$	19. 0.05 to 0.15	20. 0.21 to 0.03
21. 0.3 to 3	22. 0.5 to 10	23. 1.2 to 10	24. 6.4 to 0.8
25. $\frac{1}{2}$ to 1.5	26. $\frac{1}{4}$ to 0.75		

27. Regarding the diagram below, AC represents the length of the line segment that starts at A and ends at C.

From the diagram we see that AC = 8.



a. Find the ratio of BC to AC.
b. What is the length AE?
c. Find the ratio of DE to AE.

28. A family of four budgeted $400 for food, $100 for gas, $150 for utilities, and $750 for the house payment.

a. What's the ratio of the house payment to the food bill?
b. What is the ratio of the gas bill to the food bill?
c. What's the ratio of the utilities bills to the food bill?
d. What is the ratio of the house payment to the utilities bills?

Comparison Between Charles P. McKeague "Basic Mathematics" Textbook & Plaintiff's MAT082 Lecture Notes

Basic Mathematics Textbook. By: Charles P. McKeague, p. 289

4.2 Problem Set 289

Problem Set 4.2

Express each of the following rates as a ratio with the given units. [Examples 1, 2]

1. **Miles/Hour** A car travels 220 miles in 4 hours. What is the rate of the car in miles per hour?

2. **Miles/Hour** A train travels 360 miles in 5 hours. What is the rate of the train in miles per hour?

3. **Kilometers/Hour** It takes a car 3 hours to travel 252 kilometers. What is the rate in kilometers per hour?

4. **Kilometers/Hour** In 6 hours an airplane travels 4,200 kilometers. What is the rate of the airplane in kilometers per hour?

5. **Gallons/Second** The flow of water from a water faucet can fill a 3-gallon container in 15 seconds. Give the ratio of gallons to seconds as a rate in gallons per second.

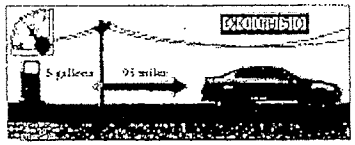
6. **Gallons/Minute** A 220-gallon drum is filled in 4 minutes. What is the rate in gallons per minute?

7. **Liters/Minute** It takes 4 minutes to fill a 66 liter gas tank. What is the rate in liters per minute?

8. **Liters/Hour** The gas tank on a car holds 60 liters of gas. At the beginning of a 6-hour trip the tank is full. At the end of the trip, it contains only 12 liters. What is the rate at which the car uses gas in liters per hour?

9. **Miles/Gallon** A car travels 90 miles on 5 gallons of gas. Give the ratio of miles to gallons as a rate in miles per gallon.

10. **Miles/Gallon** On a 384-mile trip, an economy car uses 8 gallons of gas. Give this as a rate in miles per gallon.



11. **Miles/Liter** The gas tank on a car has a capacity of 75 liters. On a full tank of gas, the car travels 525 miles. What is the gas mileage in miles per liter?

12. **Miles/Liter** A car pulling a trailer can travel 180 miles on 50 liters of gas. What is the gas mileage in miles per liter?

MAT082 Lecture Notes, Prepared by Plaintiff. p. 50

MAT 082 Basic Arithmetic Page 50 of 165 Spring 2010

29. A car travels 220 miles in 4 hours. What's the rate of the car in miles per hour?

30. A train travels 360 miles in 5 hours. What's the rate of the train in miles per hour?

31. The flow of water from a water faucet can fill a 3-gallon container in 15 seconds. Give the ratio of gallons to seconds as a rate in gallons per second.

32. A car travels 95 miles on 5 gallons of gas. Give the ratio of miles to gallons as a rate in miles per gallon.

Twenty-one people work in an office which has 15 desks and nine computers.

33. What is the ratio of desks to people?

34. What is the ratio of computers to people?

35. The ratio of females to males is 6 to 7. The class has 39 students. How many females are there?

36. The ratio of females to males is 6 to 7. The class has 39 students. How many males are there?

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Comparison Between Sullivan & Sullivan "Precalculus" Textbook & Plaintiff's MAT182 Lecture Notes

Precalculus Textbook, By: Sullivan & Sullivan, p. 394

394 CHAPTER 5 Trigonometric Functions

29. $\sin 38^\circ$	30. $\cos 14^\circ$	31. $\sin 51^\circ$	32. $\cos 30^\circ$	33. $\sec 41^\circ$	34. $\csc 55^\circ$
35. $\sin \frac{\pi}{10}$	36. $\cos \frac{\pi}{5}$	37. $\tan \frac{5\pi}{12}$	38. $\cot \frac{\pi}{18}$	39. $\sec \frac{\pi}{12}$	40. $\csc \frac{5\pi}{13}$
43. $\sin 1$	42. $\tan 1$	43. $\sin 1^\circ$	44. $\tan 1^\circ$	45. $\tan 0.3$	46. $\tan 0.1$

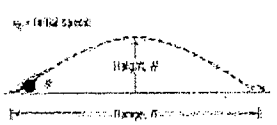
Applications and Extensions

Projectile Motion The path of a projectile fired at an inclination θ to the horizontal with initial speed v_0 is a parabola (see the figure). The range R of the projectile, that is, the horizontal distance that the projectile travels, is found by using the function

$$R(\theta) = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

where $g \approx 32.2$ feet per second per second ≈ 9.8 meters per second per second is the acceleration due to gravity. The maximum height H of the projectile is given by the function

$$H(\theta) = \frac{v_0^2 \sin^2\theta}{2g}$$



In Problems 47–50, find the range R and maximum height H. Round answers to one decimal place.

47. The projectile is fired at an angle of 45° to the horizontal with an initial speed of 100 feet per second.

48. The projectile is fired at an angle of 30° to the horizontal with an initial speed of 250 meters per second.

49. The projectile is fired at an angle of 25° to the horizontal with an initial speed of 300 meters per second.

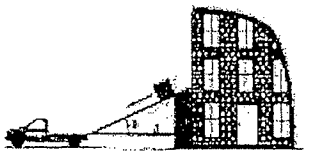
50. The projectile is fired at an angle of 50° to the horizontal with an initial speed of 200 feet per second.

51. **Inclined Plane** If friction is ignored, the time t (in seconds) required for a block to slide down an inclined plane (see the figure) is given by the function

$$t(\theta) = \sqrt{\frac{2a}{g \sin\theta \cos\theta}}$$

where a is the length (in feet) of the base and $g \approx 32$ feet per second per second is the acceleration due to gravity. How long does it take a block to slide down an inclined plane with base $a = 10$ feet when

(a) $\theta = 30^\circ$? (b) $\theta = 45^\circ$? (c) $\theta = 60^\circ$?

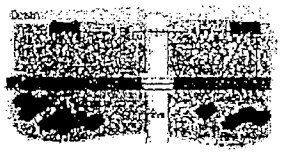


52. **Piston Engines** In a certain piston engine, the distance x (in meters) from the center of the drive shaft to the head of the piston is given by the function

$$x(\theta) = \cos\theta + \sqrt{16 + 0.5(2\cos^2\theta - 1)}$$

where θ is the angle between the crank and the path of the piston head (see the figure). Find x when $\theta = 30^\circ$ and when $\theta = 45^\circ$.

53. **Calculating the Time of a Trip** Two oceanfront homes are located R miles apart on a straight stretch of beach, each a distance of 1 mile from a paved road that parallels the ocean. Sally can jog b miles per hour along the paved road, but only s miles per hour in the sand on the beach. Because a river flows between the two houses, it is necessary to jog on the sand to the road, continue on the road, and then jog on the sand to get from one house to the other. See the figure on



MAT182 Lecture Notes, Prepared by Plaintiff, p. 22

MAT182 Trigonometry Page 22 of 71 Spring 2010

1.6 Computing the Values of Trig Functions for Special Triangles Homework

In problems 7-16, $f(\theta) = \sin\theta$ and $g(\theta) = \cos\theta$. Find the value of each expression if $\theta = 60^\circ$. Do not use a calculator.

7. $f(\theta)$ 9. $f\left(\frac{\theta}{2}\right)$ 11. $[f(\theta)]^2$ 13. $2f(\theta)$ 15. $\frac{f(\theta)}{2}$

In problems 17-27, find the exact value of each expression. *Do not use calculator.*

17. $4\cos 45^\circ - 2\sin 45^\circ$ 19. $6\tan 45^\circ - 8\cos 60^\circ$ 21. $\sec\frac{\pi}{4} + 2\csc\frac{\pi}{3}$

23. $\sec^2\frac{\pi}{6} - 4$ 25. $\sin^2 30^\circ + \cos^2 60^\circ$ 27. $1 - \cos^2 30^\circ = \cos^2 60^\circ$

In problems 29-45, use a calculator to find the approximate value of each expression. Round the answer to two decimal places.

29. $\sin 23^\circ$	31. $\tan 21^\circ$	33. $\sec 41^\circ$	35. $\sin\frac{\pi}{10}$	37. $\tan\frac{5\pi}{12}$
39. $\sec\frac{\pi}{12}$	41. $\sin 1$	43. $\sin 1^\circ$	45. $\tan 0.3$	

47. Applications and Extensions Find the range R and maximum height H rounded to 2 decimal places.

Projectile Motion The path of a projectile fired at an inclination θ to the horizontal with initial speed v_0 is a parabola (see the figure). The range R of the projectile, that is, the horizontal distance that the projectile travels, is found using the function

$$R(\theta) = \frac{2v_0^2 \sin\theta\cos\theta}{g}$$

Where $g \approx 32.2$ feet per second ≈ 9.8 meters per second is the acceleration due to gravity. The maximum height H of the projectile is given by the function

$$H(\theta) = \frac{v_0^2 \sin^2\theta}{2g}$$

Find the range R and maximum height H to two decimal places when the projectile is fired at an angle of 45° to the horizontal with an initial speed of 100 feet per second.

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Comparison Between Sullivan & Sullivan "Precalculus" Textbook & Plaintiff's MAT182 Lecture Notes

MAT182 Lecture Notes, Prepared by Plaintiff, p. 22

MAT182 Trigonometry — Page 22 of 23 — Spring 2010

1.6 Computing the Values of Trig Functions for Special Triangles Homework

In problems 7-16, $f(\theta) = \sin\theta$ and $g(\theta) = \cos\theta$. Find the value of each expression if $\theta = 60°$.

7. $f(\theta)$ 9. $f\left(\frac{\theta}{2}\right)$ 11. $[f(\theta)]^2$ 13. $2f(\theta)$ 15. $\frac{f(\theta)}{2}$

In problems 17-27, find the exact value of each expression. *Do not use calculator.*

17. $4\cos 45° - 2\sin 45°$ 19. $6\tan 45° - 8\cos 60°$ 21. $\sec\frac{\pi}{4} + 2\csc\frac{\pi}{3}$

23. $\sec^2\frac{\pi}{6} - 4$ 25. $\sin^2 30° + \cos^2 60°$ 27. $1 - \cos^2 30° - \cos^2 60°$

In problems 29-45, use a calculator to find the approximate value of each expression. Round the answer to two decimal places.

29. $\sin 25°$ 31. $\tan 21°$ 33. $\sec 41°$ 35. $\sin\frac{\pi}{10}$ 37. $\tan\frac{5\pi}{12}$

39. $\sec\frac{\pi}{12}$ 41. $\sin 1$ 43. $\sin 1°$ 45. $\tan 0.3$

47. Applications and Extensions Find the range R and maximum height H rounded to 2 decimal places.

Projectile Motion The path of a projectile fired at an inclination θ to the horizontal with initial speed v_0 is a parabola (see the figure). The range R of the projectile, that is, the horizontal distance that the projectile travels, is found using the function

$$R(\theta) = \frac{2v_0^2 \sin\theta\cos\theta}{g}$$

Where $g = 32.2$ feet per second $= 9.8$ meters per second is the acceleration due to gravity. The maximum height H of the projectile is given by the function

$$H(\theta) = \frac{v_0^2 \sin^2\theta}{2g}$$

Find the range R and maximum height H to two decimal places when the projectile is fired at an angle of 45° to the horizontal with an initial speed of 100 feet per second.

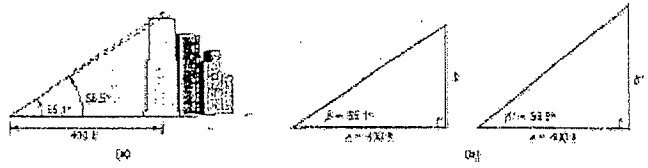
GARRISON 2009

Precalculus Textbook, By: Sullivan & Sullivan, p. 393

SECTION 5.3 Computing the Values of Trigonometric Functions of Acute Angles 393

...found to be 55.1° and the angle of elevation to the top of the statue is 56.5°. See Figure 36(a). What is the height of the statue?

Figure 36



Solution Figure 36(b) shows two triangles that replicate Figure 36(a). The height of the statue (Ceres will be $b' - b$. To find b and b', we refer to Figure 36(b).

$$\tan 55.1° = \frac{b}{400} \qquad \tan 56.5° = \frac{b'}{400}$$

$$b = 400 \tan 55.1° \approx 573.39 \qquad b' = 400 \tan 56.5° \approx 604.33$$

The height of the statue is approximately $604.33 - 573.39 = 30.94$ feet ≈ 31 feet.

NOW WORK PROBLEM 61.

5.3 Assess Your Understanding

Concepts and Vocabulary

1. $\tan\frac{\pi}{4} + \sin 30° =$ _____.
2. Using a calculator, $\sin 2° =$ _____ rounded to two decimal places.
3. *True or False:* Exact values can be found for the trigonometric functions of 60°.
4. *True or False:* Exact values can be found for the sine of any angle.

Skill Building

5. Write down the exact value of each of the six trigonometric functions of 45°.
6. Write down the exact value of each of the six trigonometric functions of 30° and of 60°.

In Problems 7–16, $f(\theta) = \sin\theta$ and $g(\theta) = \cos\theta$. Find the exact value of each expression if $\theta = 60°$. Do not use a calculator.

7. $f(\theta)$ 8. $g(\theta)$ 9. $f\left(\frac{\theta}{2}\right)$ 10. $g\left(\frac{\theta}{2}\right)$ 11. $[f(\theta)]^2$
12. $[g(\theta)]^2$ 13. $2f(\theta)$ 14. $2g(\theta)$ 15. $\frac{f(\theta)}{2}$ 16. $\frac{g(\theta)}{2}$

In Problems 17–38, find the exact value of each expression. Do not use a calculator.

17. $4\cos 45° - 2\sin 45°$ 18. $2\sin 45° + 4\cos 30°$ 19. $6\tan 45° - 8\cos 60°$
20. $\sin 30° \cdot \tan 60°$ 21. $\sec\frac{\pi}{4} + 2\csc\frac{\pi}{3}$ 22. $\tan\frac{\pi}{4} + \cot\frac{\pi}{4}$
23. $\sec^2\frac{\pi}{6} - 4$ 24. $4 + \tan^2\frac{\pi}{3}$ 25. $\sin^2 30° + \cos^2 60°$
26. $\sec^2 60° - \sin^2 45°$ 27. $1 - \cos^2 30° - \cos^2 60°$ 28. $1 + \cot^2 30° - \csc^2 45°$

EXHIBIT 8

Chapters for Basic Arithmetic

1 Numbe Operations

2 F tions

3 G metry

4 Inte s (S.N.)

5 Decima nd Percents

6 Me Graphs

Primary emphasis placed on fundamental operations with whole numbers, fractions, decimals, integers, and rational numbers; proportions, and percentages. Other topics include representations of data, geometric figures, and measurement.

Thank you

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Mrs. Elena Sung
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MAT 082 Spring 2010 Outline

Basic Arithmetic

Final Exam will be May 13, 2010, in Room B210
8:30-9:45am Thursday

Chapter 1 – Numbers and Operations

1.1 *Types of Numbers and Vocabulary*

Day 1: *Whole Number, Mixed Number, Negative Number, Positive Decimal, Negative Decimal, Improper Fraction, Proper Fraction, Natural Number, Integer*

The following vocabulary will be covered in the next eight days as follows:

Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9
A-C	D-F	I-M	N-R	R-W	Symbols	Complex	Powers
						Even/odd	Roots

Math Vocabulary Words

Absolute Value is the distance from zero of a number inside vertical bars.

$|-7|$ is 7 because -7 is 7 units away from zero

Addends are numbers being added.

$$\begin{array}{r} \textbf{addend} \\ +\ \textbf{addend} \\ \hline \textit{sum} \end{array}$$

The **Associative Property** has to do with grouping using parentheses (2+5), brackets [2+5], or braces {2+5}. The associative property says that when *adding or multiplying* you can group different numbers without changing the value of the answer.
Example: (2+5)+8 is the same as 2+(5+8)

The **average** of some numbers is found by adding all the numbers and dividing the answer by however many numbers were added. Another word for *average* is the *mean*.
Example: the average of 2, 5, and 8 is (2+5+8) divided by 3 which is 15/3 or 5

The **base** in the number 3^2 is the 3. 3^2 means 3 times 3 which is 9.

Commutative Property has to do with changing the order of adding or multiplying. The commutative property says that changing the order of numbers does not change the value of the answer.
Example: 2+5+8 is the same as 5+8+2

Composite numbers are numbers that can be written as the multiplication of 3 or more numbers. Example: 6 is (1)(2)(3) so 6 is a composite number However, 5 is not composite because 5 is (1)(5). Five is called a prime number.
FYI: the number "1" is the only number which not prime and not composite either.

A **Constant** is a number and a variable is a letter.
Example: 7 is a constant and y is a variable

Cubed means multiplied times itself *three* times.
Example: 5 cubed is written 5^3 and this means (5)(5)(5) which is 125

Distribute means to *multiply more than one time.*
Example: $3(4+5)$ means to multiply the 3 times the 4 and the 3 times the 5, then add those two numbers.

$$3(4+5) = 3(4) + 3(5) = 12+15 = 27$$

The **denominator** is the number written at the bottom of a fraction and this is the number that gives the fraction its name. The top number is the numerator which tells you how many you have.

Example: In $\frac{20}{5}$ the 5 is the denominator. $\frac{\textbf{numerator}}{\textbf{denominator}}$ $\frac{\leftarrow \textit{dividend}}{\leftarrow \textit{divisor}}$

Dimensional analysis is the procedure to change from one unit of measurement to another by using fractions whose value is "1" like $\frac{1\ foot}{12\ inches}$ or $\frac{3\ feet}{1\ yard}$ or $\frac{5280\ feet}{1\ mile}$.

Example: Change 5 feet into inches.

$$5 \text{ feet} = \frac{5\ ft.}{1} \cdot \frac{12\ in.}{1\ ft.} = 5 \cdot 12 \text{ in.} = 60 \text{ inches}$$

The **dividend** is the number you are dividing *into.*
The **divisor** is the number you are *dividing by.*
The **quotient** is *the answer* of a division problem.

$$\textbf{divisor}\overline{)\textbf{dividend}}\quad \textbf{quotient}$$

Evaluate means give the value. Example: Evaluate $x+2$ when the x is 7.
Answer: The value of $x+2$ is 9.

An **Exponent** is the small number telling you how many times to multiply the base. (The exponent is written above and to the right of the larger number called the base)

3^4 (base → 3, ←exponent 4) i.e. *3 to the 4th power* is (3)(3)(3)(3) which is 81

A **factor** is something multiplied times something else.

Examples: 9×5 $9\cdot5$ $(9)(5)$ $a\times b$ $C\cdot D$ $(x)(y)$ $5x$

To **factor** means "write as multiplication"
Example: Factor 45 means to write it as (5)(9) or (3)(15) or (3)(3)(5)

An **improper fraction** has the top number (numerator) larger than the bottom number or the same size as the bottom number.

Examples: $\frac{5}{5}$ $\frac{13}{7}$

Index $\sqrt[\text{index}]{\text{radicand}}$ Example: $\sqrt[3]{64}$ 3 is the index

$\sqrt{16}$ is $\sqrt[2]{16}$ 2 is the index

Integers are all the positive numbers, the negative numbers, and the zero.
Integers are ... $-6, -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5$, ...

Invert the fraction means to flip numerator and denominator.

Example: If you invert $\frac{2}{3}$ you get $\frac{3}{2}$ which is also called the "reciprocal".

LCD stands for lowest common denominator.
LCM stands for least common multiple. LCM means the same as LCD.

Example: The LCM for 2, 5, 6 is 30 The LCD for $\frac{1}{2}+\frac{3}{5}+\frac{1}{6}$ is 30.

Each number can be multiplied to get 30: $2(15)=30$ $5(6)=30$ $6(5)=30$

Also, 30 can be divided evenly by each number: $\frac{30}{2}$ $\frac{30}{5}$ $\frac{30}{6}$

The **mean** of some numbers is found by adding all the numbers and dividing the answer by however many numbers were added. Another word for *mean* is the *average*.
Example: 5 is the *mean* of 2, 5, and 8 since (2+5+8) divided by 3 is 15/3 or 5

The **median** is the middle number in a list of numbers listed in order of size.
Example: 8 is the *median* of the numbers 2, 2, 3, 6, 7, **8**, 19, 32, 39, 40, 75

The **minuend** is the number you are subtracting from.

minuend
– subtrahend
difference

A **mixed number** is a whole number with a fraction like $2\frac{3}{8}$.
You can change a mixed number to a fraction by multiplying 8 times the 2 then add the 3 so you get the fraction $\frac{19}{8}$.

The **mode** of some numbers is the number that is repeated the most times.
For example: 28 is the *mode* for 3, 9, 14, 17, 3, 28, 19, 28, 6, 5, 28

Natural numbers are the numbers 1, 2, 3, 4, 5, 6, 7, 8, . . .
They are also called "counting numbers."

Negative numbers are the numbers less than zero.

The **numerator** is the number written at the top of a fraction and this is the number that tells us how many you have. The bottom number is the denominator which tells you the name of the fraction.

Example: In $\frac{20}{5}$ the 20 is the numerator. $\frac{\textbf{numerator}}{\textbf{denominator}}$ $\frac{\leftarrow \textit{dividend}}{\leftarrow \textit{divisor}}$

Positive numbers are the numbers greater than zero.

The **Power** of a number is the *exponent*. 3^2 is read "3 to the second power."
The "2" is the power of the "3."

A **prime number** can be divided by no other number except 1 and itself.
Examples of prime numbers: 2, 3, 5, 7
FYI: the number "1" is the only number which not prime and not composite either.

Prime factorization is factoring by using only prime numbers

A **product** is the answer to multiplication.

factor *times* **factor** = **product**

$$\begin{array}{r} \textbf{factor} \\ \times\ \textbf{factor} \\ \hline \textit{product} \end{array}$$

A **proper fraction** has the top number smaller than the bottom number.
Example of a proper fraction: $\frac{6}{7}$

A **quotient** is the answer to division.

$$\frac{\text{numerator}}{\text{denominator}} = \frac{\textit{dividend}}{\textit{divisor}} = \textit{quotient}$$

$$\begin{array}{r} \textbf{quotient} \\ \textbf{divisor}\overline{)\textbf{dividend}} \end{array}$$

The **radicand** is the part written inside the radical.

$\sqrt[\text{index}]{\textbf{radicand}}$ $\sqrt[3]{8}$ "8" is the radicand

Rational numbers are numbers that have *repeating decimals.*

Examples: $\frac{1}{2}$ is $.500\overline{0}$ $\frac{3}{8}$ is $.37500\overline{0}$ $\frac{2}{3}$ is $.666\overline{6}$ $7.512323\overline{23}$

These are not rational numbers

(i.e. irrational numbers): $\sqrt{5}$ π $.573733733373333\cdots$

The **reciprocal** is the inverted or flipped fraction.

Example: The reciprocal of $\frac{4}{8}$ is $\frac{8}{4}$

The **square root** is represented by this radical symbol $\sqrt{\ }$ $\sqrt{radicand}$

The part inside is called the radicand. The square root of 49 = $\sqrt{49}$ = 7.

The square root of a number is the number which times itself is the radicand.

Example: $\sqrt{49}$ is 7 because 7 times 7 is 49 which is the radicand

Square numbers are the result of multiplying a number times itself. Examples of squared numbers are: 0, 1, 4, 9, 25, 36, 49, 64

The **subtrahend** is the number that is being subtracted in a subtraction problem.

minuend
− subtrahend
difference

A **variable** is a letter and a constant is a number.

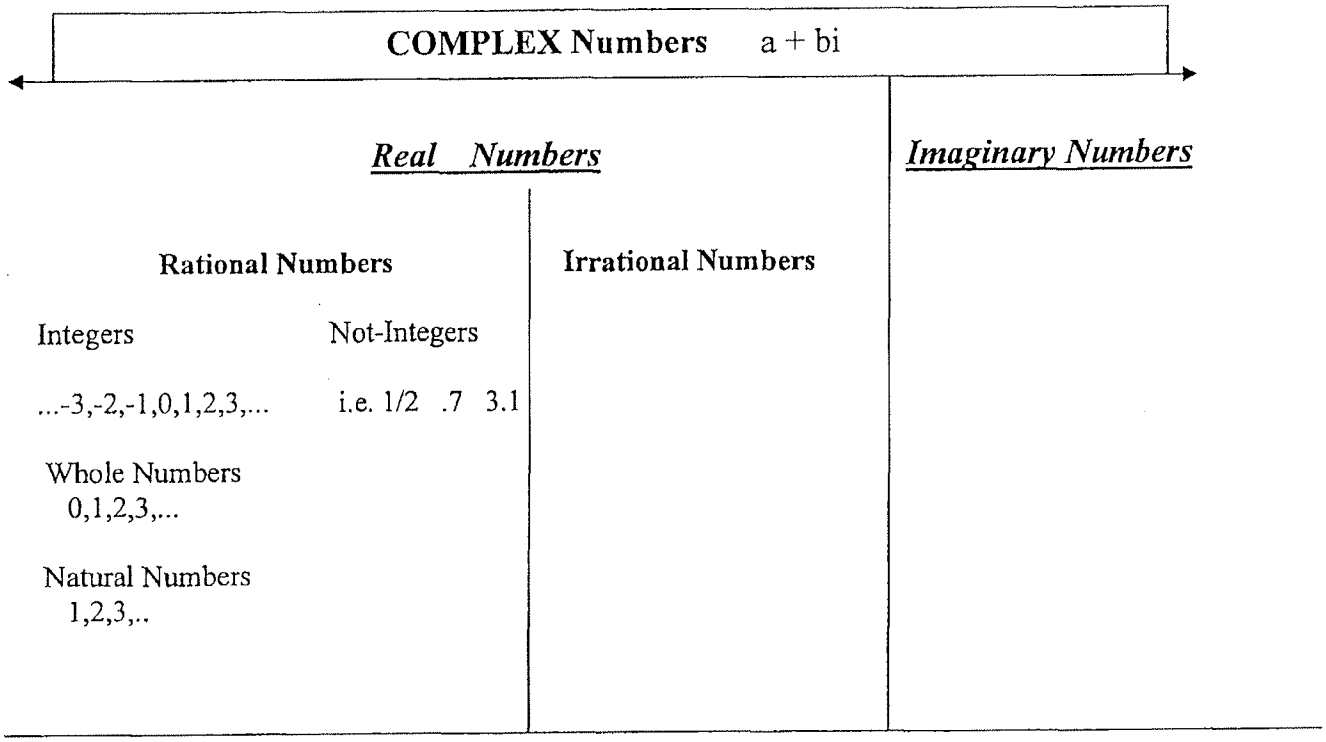
Example: 7 is a constant and y is a variable

Whole numbers are the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, . . .

Notice that "0" is a whole number but it is not a "natural number."

Math Vocabulary in Symbols

<	**is less than**	≤	**is less than or equal to**
>	**is greater than**	≥	**is greater than or equal to**
=	**is equal to**	≈	**is approximately equal to**
| |	**absolute value**	!	**factorial**
⟌	**division**	π	**pi ≈3.14 or $\frac{22}{7}$**
$\sqrt{\ }$	**the square root of**	e	**≈2.718**
$\sqrt[3]{\ }$	**the cube root of**	i	$\sqrt{-1}$ **imaginary number**
$\sqrt[4]{\ }$	**the fourth root of**	∥	**is parallel to**
$\sum$	**summation (add)**	⊥	**is perpendicular to**



Even Numbers are numbers ending with 0, 2, 4, 6, or 8. They can be divided evenly by 2.

Odd Numbers are the rest of the numbers: 1, 3, 5, ___, ___, ___ ...

List the first 6 even numbers _____ _____ _____ _____ _____ _____

List the first 6 odd numbers _____ _____ _____ _____ _____ _____

List the first 6 prime numbers _____ _____ _____ _____ _____ _____

List the first 6 composite numbers _____ _____ _____ _____ _____ _____

Properties of Real Numbers

If two fractions are added together, do you always get a fraction?

If two real numbers are added together, do you always get a real number? Closure Property for Addition

If two fractions are multiplied, do you always get a fraction?

If two real numbers are multiplied, do you always get a real number? Closure Property for Muliplication

Can you add real numbers in any order? like 3+5+9 or 5+3+9 Commutative Property of Addition
Is the answer the same with both additions?

Can you multiply real numbers in any order? like 3(5)9 or 5(9)(3) Commutative Property of Addition

Can you subtract real numbers in any order? like $9-5$ or $5-9$ Are the answers the same?

Can you divide real numbers in any order? like $8\div 2$ or $2\div 8$ Are the answers the same?

What is 12 divided by 3? How do you check this division?

What is 0 divided by 5

What is 12 **divided by 0**?

What number can you add to a number and the first number is the answer? Zero—the Additive Identity

What can you multiply times a number and the first number is the answer? One—the Multiplicative Identity

What do you add to a number to get the answer zero? Its opposite—The Additive Inverse

What do you multiply times a number to get the answer one? Its inverse—The Multiplicative Inverse

However, the number zero does not have a multiplicative inverse.

What happens when you multiply a number times one? Why? the Multiplicative Identity

What happens when you add zero to a number? Why? the Additive Identity

What happens when you multiply a number times zero? Why? the Multiplication Property of Zero

What happens when you multiply a number times one? Why? the Multiplicative Identity

---

The last vocabulary words we will talk about are numbers called "squares," "cubes," square roots, and cube roots.

	Square Numbers	Cubes		Square roots & Cube Roots	
0^2	**0**	**0**	0^3	$\sqrt[2]{0} = \mathbf{0}$	$\sqrt[3]{0} = \mathbf{0}$
1^2	**1**	**1**	1^3	$\sqrt[2]{1} = \mathbf{1}$	$\sqrt[3]{1} = \mathbf{1}$
2^2	**4**	**8**	2^3	$\sqrt[2]{4} = \mathbf{2}$	$\sqrt[3]{8} = \mathbf{2}$
3^2	**9**	**27**	3^3	$\sqrt[2]{9} = \mathbf{3}$	$\sqrt[3]{27} = \mathbf{3}$
4^2	**16**	**64**	4^3	$\sqrt[2]{16} = \mathbf{4}$	$\sqrt[3]{64} = \mathbf{4}$
5^2	**25**	**125**	5^3	$\sqrt[2]{25} = \mathbf{5}$	$\sqrt[3]{125} = \mathbf{5}$
6^2	**36**	**216**	6^3		
7^2	**49**	**343**	7^3		
8^2	**64**	**512**	8^3		
9^2	**81**	**729**	9^3		
10^2	**100**	**1,000**	10^3		
11^2	**121**				
12^2	**144**				
13^2	**169**				

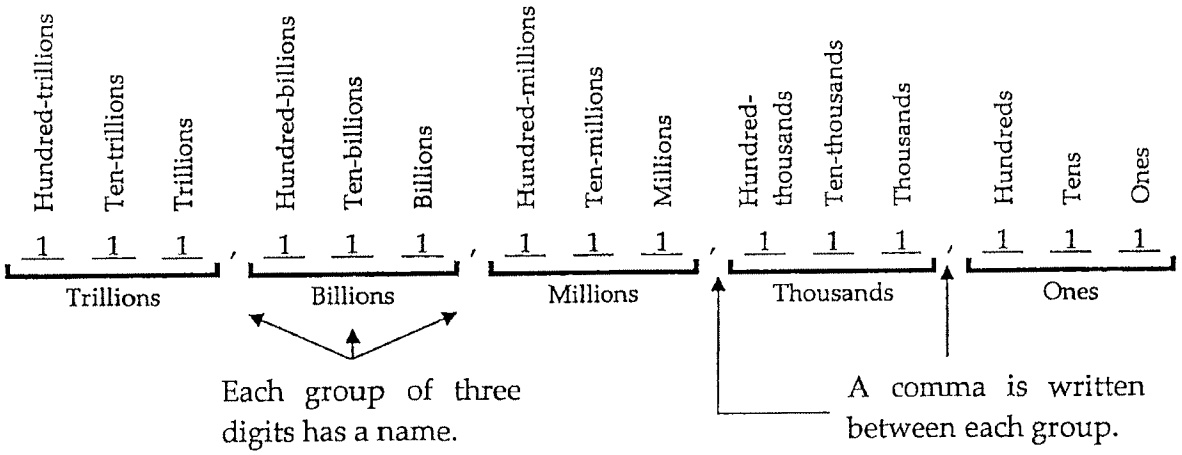
1.1 *Types of Numbers and Vocabulary* Homework

Identify the number as *Whole Number, Mixed Number, Negative Number, Positive Decimal, Negative Decimal, Improper Fraction, Proper Fraction, Natural Number, Integer*

1. 15	2. $8\frac{3}{4}$	3. 0	4. 3.781	5. 83,001
6. −8	7. $\frac{7}{16}$	8. $\frac{9}{5}$	10. 33.7	11. −5
12. 457	13. $\frac{8}{5}$	14. $1\frac{3}{4}$	15. −14.1	16. 5.8
17. $\frac{5}{4}$	18. $\frac{1}{10}$	19. 362,049	20. 01	21. $7\frac{7}{8}$
22. 33.7	23. 24.	25. π	26. 75,039	27. $\frac{1}{3}$
28. −87	29. 6.49	30. −.5	31. $2\frac{7}{10}$	32. $\frac{15}{8}$

33. Which are even numbers? 3, 14, 9, -8, 71, 50, π, e, $4\frac{1}{2}$, $\sqrt{25}$, $|-7|$, $\sqrt{36}$

1.2 Whole Number Place Value Chart



1.2 Place Value Homework

Identify the *place value* of the digit **2** in each number.

1. 61,284
2. 82,110
3. 284,100
4. 823,415
5. 725,837,166
6. 44,265,199
7. 253,045,701,000
8. 823,000,419,567

Name the place value for each 0 in this number: 302,016,450,098,570

9. The 1st zero _____
10. The 4th zero _____
11. The 2nd zero _____
12. The 5th zero _____
13. The 3rd zero _____

Name the place value for each 0 in this number: 810,704,069,809,035

14. The 1st zero _____
15. The 4th zero _____
16. The 2nd zero _____
17. The 5th zero _____
18. The 3rd zero _____

Write each number in words.

19) 8421 20) 1936 21) 46,205 22) 75,089 23) 3,064,801 24) 7,900,408

Translate the words into numbers.

25. Forty-six thousand, eight hundred five.
26. Seventy-nine thousand, forty-six
27. Five million, six hundred thousand, eighty-two
28. One million, thirty thousand five
29. Two hundred seventy-one million, nine hundred thousand
30. Three hundred eleven million, four hundred

Which of the numbers $\{0, \frac{2}{3}, -4\frac{3}{5}, -\frac{12}{3}, \sqrt{\frac{81}{4}}, \sqrt{10}, \pi, 72, e, i, -65, 5i\}$ are:

31. Natural numbers _____
32. Integers _____
33. Rational numbers _____
34. Whole Numbers _____

1.3 Rounding Numbers

Steps for rounding numbers

Step 1 Locate the number that is being rounded. Replace the numbers **after** that number with zeros.

Step 2 Look **only** at the **first** number you are changing to zero.

If this number is **4 or less**, the rounded number **stays the same.**

If this number is **5 or more**, add 1 to the rounded number.

Step 3 Use the ≈ symbol to indicate that the rounded number is now an approximation (close, but *not exact*).

Example 1: Round 14.39652 to the nearest thousandth. (*Is it closer to 14.396 or 14.397?*)

1.3 *Rounding Numbers* Homework

Round to the nearest ten:

1) 45	**2)** 85	**3)** 661	**4)** 123

Round to the nearest hundred:

5) 16,462	**6)** 12,799	**7)** 823,402	**8)** 701,529

Round to the nearest thousand:

9) 38,431	**10)** 117,011	**11)** 12,577	**12)** 671,529

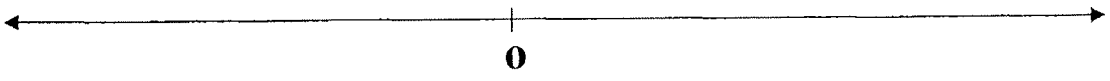
Round to the nearest hundred thousand:

13) 5,254,423	**14)** 1,395,999	**15)** 9,007,601	**16)** 3,116,201

1.4 Basic Operations: Adding

There are 4 basic operations: Addition, Subtraction, Multiplication, and Division.

negative numbers on this side *positive numbers on this side*



This line is called the number line.
It has all the negative numbers on the left of zero and all the positive numbers on the right.

Numbers which are larger than zero are called positive numbers.
Positive numbers can be written with a "+" sign in front of them or without any sign at all.
Saving 4 dollars and saving 11 dollars is like adding two positive numbers. This would be a savings of 15 dollars. In numbers this looks like this: $\begin{array}{r} +4+11 \\ +15 \end{array}$ or $\begin{array}{r} 4+11 \\ 15 \end{array}$

Negative numbers are smaller than zero and they must be written with a minus sign in front.
If you spend 8 dollars at one store, then spend 6 dollars at another store, you spent 14 dollars.
In numbers this looks like this: $\begin{array}{r} -6-8 \\ -14 \end{array}$

Add these positive numbers: $13+8+2+21+6$

Add these negative numbers: $-13-8-2-21-6$

1.4 Adding Homework

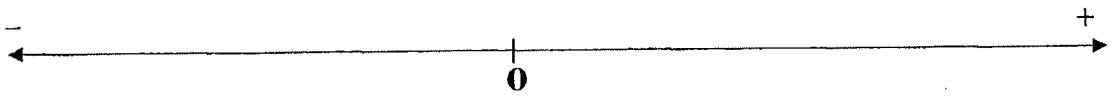
Perform the following addition.

1. $\begin{array}{r} 41 \\ \underline{28} \end{array}$
2. $\begin{array}{r} -64 \\ \underline{-39} \end{array}$
3. $\begin{array}{r} 56 \\ \underline{38} \end{array}$
4. $\begin{array}{r} -21 \\ \underline{-17} \end{array}$
5. $\begin{array}{r} 83 \\ \underline{75} \end{array}$
6. $\begin{array}{r} 138 \\ \underline{96} \end{array}$
7. $\begin{array}{r} 256 \\ \underline{209} \end{array}$
8. $\begin{array}{r} -384 \\ \underline{-123} \end{array}$
9. $\begin{array}{r} 489 \\ \underline{261} \end{array}$
10. $\begin{array}{r} 341 \\ \underline{96} \end{array}$
11. $\begin{array}{r} -482 \\ \underline{-82} \end{array}$
12. $\begin{array}{r} 362 \\ \underline{94} \end{array}$
13. $\begin{array}{r} -416 \\ \underline{-329} \end{array}$
14. $\begin{array}{r} -3684 \\ \underline{-2318} \end{array}$
15. $\begin{array}{r} -4829 \\ \underline{-3264} \end{array}$
16. $\begin{array}{r} 4,826,378 \\ \underline{2,918,436} \end{array}$
17. $\begin{array}{r} 3,821,987 \\ \underline{2,943,998} \end{array}$
18. $\begin{array}{r} 2,381,236 \\ \underline{493,648} \end{array}$
19. $\begin{array}{r} -3,472,831 \\ \underline{-2,574,283} \end{array}$
20. $\begin{array}{r} 5,000,000 \\ \underline{4,872,381} \end{array}$
21. $\begin{array}{r} -4,000,000 \\ \underline{-278,923} \end{array}$
22. $\begin{array}{r} 3,000,000 \\ \underline{2,872,917} \end{array}$
23. $\begin{array}{r} 6,000,000 \\ \underline{378,293} \end{array}$

1.5 Basic Operations: Subtraction *This is adding numbers with different signs.*

A plus sign (+) means positive or plus. A negative (−) sign means minus or negative.

←left is the negative direction right is the positive direction→



Adding on the number line: *This is adding numbers which have the same sign.*

Consider $+3+5$. On the number line you start at 0 then go to the right 3 spaces, then go to the right 5 more spaces. This puts you at +8.

Consider $-2-4$. On the number line you start at 0 then go to the *left* 2 spaces, then go to the left 4 more spaces. This puts you at −6.

Adding numbers with opposite signs *is called subtraction.*

Subtraction is the opposite operation of addition. This is why the "+" direction is to the right, and the "−" direction is the opposite way, to the left.

Consider $+8-5$. On the number line you start at 0 then go to the *right* 8 spaces, then go the *left* 5 spaces. This puts you at +3.

Examples: Add the following numbers.

+15−10 −15+10 +17−8 −17+8 +500−200

Examples with larger numbers: Add the following numbers.

+51	+189	+5728
−21	−122	−2345

1.5 Basic Operations: Subtraction Homework

1. $+41 - 28$
2. $+64 - 39$
3. $-56 + 38$
4. $+21 - 17$
5. $+83 - 75$
6. $138 - 96$
7. $256 - 209$
8. $+384 - 123$
9. $+89 - 261$
10. $+341 - 96$
11) $482 - 82$
12) $-362 + 94$
13) $-416 + 329$
14) $+3684 - 2318$
15) $-4829 + 3264$
16. $+4,826,378 - 2,918,436$
17. $-3,821,987 + 2,943,998$
18. $+2,381,236 - 493,648$
19. $-3,472,831 + 2,574,283$
20. $+5,000,000 - 4,872,381$
21. $4,000,000 - 278,923$
22. $-3,000,000 + 2,872,917$
23. $+6,000,000 - 378,293$

24. Mario paid $885 for his motorcycle while Bob paid $1348 for his. How much more did Bob pa y than Mario for his motorcycle?

25. Joaquin shot a 132 at the golf course last week while his wife shot an 87. What was the differen ce in their scores.

26. Maria bowled a 207 last evening in the Bowling Tournament. The tournament was won with a score o f 281. What was the difference between Maria's score and the winning score?

27. Carlos was looking at a boat which cost $1595. He was told by the store manager that he could save $475 b y waiting until the boat was on sale. If Carlos waited for the sale, how much would the boat cos t him then?

28. Mrs. Trujillo placed a meat order for 300 pounds of beef. When she went to pick it up she was told she could onl y have 175 pounds. How much difference was there between what Mrs. Trujillo wanted and what she actually received?

29. Jim made a 78 on his first math exam while Francisco made a 92. How many points better did Francisco do than Jim?

30. Amalia lived 438 miles from Denver while Veronica lived only 289 miles. How much closer did Veronica live to D enver than Amalia?

1.6 Basic Operations: Multiplication

MULTIPLICAND: The number that is being multiplied.

MULTIPLIER: The number doing the multiplying.

PRODUCT: The answer we get in a multiplication.

FACTORS: The numbers being multiplied together (the multiplicand and the multiplier).

6 MULTIPICAND
x 8 MULTIPLIER
48 PRODUCT

$6 \times 8 = 6 \cdot 8 = (6)(8) = 48$

Multiplication is indicated by using parentheses ( ) ( ) or a dot "·". Here is an example of the dot between the numbers $4 \cdot 3$ and the use of parenthesis (4) (3).

Multiplication is really repeated addition. That is, (6) (3) is really $3 + 3 + 3 + 3 + 3 + 3 = 18$.

Multiplication Tables from 1 to 13

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	2	3	4	5	6	7	8	9	10	11	12	13
2	2	4	6	8	10	12	14	16	18	20	22	24	26
3	3	6	9	12	15	18	21	24	27	30	33	36	39
4	4	8	12	16	20	24	28	32	36	40	44	48	52
5	5	10	15	20	25	30	35	40	45	50	55	60	65
6	6	12	18	24	30	36	42	48	54	60	66	72	78
7	7	14	21	28	35	42	49	56	63	70	77	84	91
8	8	16	24	32	40	48	56	64	72	80	88	96	104
9	9	18	27	36	45	54	63	72	81	90	99	108	117
10	10	20	30	40	50	60	70	80	90	100	110	120	130
11	11	22	33	44	55	66	77	88	99	110	121	132	143
12	12	24	36	48	60	72	84	96	108	120	132	144	156
13	13	26	39	52	65	78	91	104	117	130	143	156	169

1.6 Multiplication Homework

Multiply

1. $\begin{array}{r} 38 \\ \times\ 2 \end{array}$
2. $\begin{array}{r} 28 \\ \times\ 6 \end{array}$
3. $\begin{array}{r} 22 \\ \times\ 7 \end{array}$
4. $\begin{array}{r} 73 \\ \times\ 6 \end{array}$
5. $\begin{array}{r} 68 \\ \times\ 7 \end{array}$
6. $\begin{array}{r} 49 \\ \times\ 8 \end{array}$

7. $32(9)$
8. $18(8)$
9. $35(5)$
10. $42(8)$
11. $39(4)$
12. $28(3)$

13. $29 \cdot 7$
14. $82 \cdot 9$
15. $69 \cdot 8$
16. $30 \cdot 6$
17. $78 \cdot 42$
18. $39 \cdot 21$

Multiply

19. $\begin{array}{r} 62 \\ \times\ 42 \end{array}$
20. $\begin{array}{r} 21 \\ \times\ 78 \end{array}$
21. $\begin{array}{r} 37 \\ \times\ 48 \end{array}$
22. $\begin{array}{r} 28 \\ \times\ 73 \end{array}$
23. $\begin{array}{r} 21 \\ \times\ 98 \end{array}$
24. $\begin{array}{r} 37 \\ \times\ 43 \end{array}$

25. $\begin{array}{r} 31 \\ \times\ 8 \end{array}$
26. $\begin{array}{r} 723 \\ \times\ 4 \end{array}$
27. $\begin{array}{r} 138 \\ \times\ 6 \end{array}$
28. $\begin{array}{r} 213 \\ \times\ 18 \end{array}$
29. $\begin{array}{r} 315 \\ \times\ 13 \end{array}$
30. $\begin{array}{r} 142 \\ \times\ 18 \end{array}$

31. $\begin{array}{r} 168 \\ \times\ 12 \end{array}$
32. $\begin{array}{r} 287 \\ \times\ 32 \end{array}$
33. $\begin{array}{r} 361 \\ \times\ 212 \end{array}$
34. $\begin{array}{r} 203 \\ \times\ 142 \end{array}$
35. $\begin{array}{r} 368 \\ \times\ 102 \end{array}$
36. $\begin{array}{r} 368 \\ \times\ 213 \end{array}$

37. $382(726)$
38. $398(219)$
39. $364(13)$
40. $3462(213)$

41. Ernesto went to the market to buy steaks which cost 3 dollars per pound. When he had the steak weighed, it was 12 pounds. How much did the steaks cost?

42. Gonzalez Canning purchased 23 canning machines which cost $1683 each. How much did they pay for all of the machines?

43. The Buena Vista Hotel has 412 rooms. If each room rented for $14 per night, how much would the hotel make if they had a night when all the rooms were filled?

44. The Bravo Carpet Company would like to carpet your entire house at a cost of 7 dollars per square yard. It was found that your house contained 138 square yards. how much would you have to pay to have the carpeting done?

45. Ofelia's Catering Company is planning a paving project in your neighborhood. The county says it will cost each family $185. There are 175 families in your neighborhood. how much will the project cost?

46. Ana Lisa bought 498 dresses wholesale for her dress shop. The company charged her 12 dollars per dress. How much did Ana Lisa pay for the dresses?

1.7 Long Division

Zero divided by any number is zero.

$\frac{0}{8} = 0$ because $8(0) = 0$ $\quad 0 \div 132 = 0$ $\quad \frac{0}{12} = 0$ $\quad \frac{0}{9} = 0$

zero divided by 8 is 0

Division by zero is undefined $\quad \frac{N}{O}$ $\quad \frac{8}{0}$ $\quad 132 \div 0$ $\quad \frac{12}{0}$ $\quad \frac{9}{0}$

These are all called undefined quantities.

QUOTIENT: The answer to a division problem.

DIVISOR: The number doing the dividing. **DIVIDEND:** The number that is being divided.

Division can be indicated in three different ways.

This is called
Long Division

$$\begin{array}{r} 4 \\ 3\overline{)12} \\ \underline{12} \\ \end{array}$$

4 quotient; divisor 3, dividend 12

Using the
Division Symbol

$12 \div 3 = 4$

dividend ÷ divisor = quotient

Fraction form

$\frac{12}{3} = 4$

dividend / divisor = quotient

Steps for Long Division

The division algorithm
1st. ***Divide***
2nd ***Multiply***
3rd ***Change the sign***
4th ***Bring down the next term***
Repeat the steps

$342 \div 12$

1. Set up the problem so that 12 is outside and 342 is inside.

$12\overline{)342}$

2. Does 12 divide into 3? No.

3. Does 12 ***divide*** into 34? Yes, 2 times.
   Place the 2 above the 4.

$$\begin{array}{r} 2 \\ 12\overline{)342} \\ 24 \end{array}$$

4. ***Multiply*** 2 x 12 = 24 and place the product 24 below 34.

5. ***Change the sign*** *of* 24 to minus 24. Add +34 and −24.

6. ***Bring down*** the 2 in the dividend to become the 102.

7. How many times will 12 ***divide*** into 102?
   (8 times since 8x12 = 96.)

8. Place the 8 above the 2 in the divisor.

9. ***Multiply*** the 8 times the 12 and place the product under the 102.

10. ***Change the sign*** *of* the 96 to minus 96. Subtract.

The division algorithm:
1st. ***Divide***
2nd ***Multiply***
3rd ***Change the sign***
4th ***Bring down next term***
Repeat the steps

Since there is no number to bring down and 12 will not divide into 6, we have a remainder of 6. Our final problem looks like this.

or $28\frac{6}{12}$ or

The main thing to keep in mind here is to continue the dividing and subtracting until you can't divide any longer and there is nothing else to bring down.

1.7 Long Division Homework

Divide from memory or use long division.

1. $32 \div 8$	2. $24 \div 4$	3. $36 \div 6$	4. $54 \div 9$
5. $72 \div 8$	6. $64 \div 8$	7. $42 \div 6$	8. $63 \div 9$
9. $15 \div 5$	10. $81 \div 9$	11. $92 \div 2$	12. $42 \div 7$
13. $420 \div 7$	14. $0 \div 9$	15. $121 \div 11$	16. $144 \div 12$
17. $3 \div 0$	18. $400 \div 20$	19. $800 \div 25$	20. $650 \div 13$
21. $387 \div 8$	22. $492 \div 9$	23. $682 \div 11$	24. $893 \div 12$
25. $312 \div 13$	26. $1872 \div 9$	27. $689 \div 42$	28. $938 \div 81$
29. $298 \div 13$	30. $1647 \div 3$	31. $8461 \div 36$	32. $8136 \div 142$
33. $436 \div 12$	34. $893 \div 11$	35. $3846 \div 132$	36. $847 \div 12$
37. $693 \div 9$	38. $1382 \div 12$	39. $841 \div 89$	40. $4632 \div 13$

41. Three men received $210 for a job they did together. If they each received equal amounts, how much did each man receive for the work done?

42. Ajax Car Rental purchased 38 cars for a total of $144,400 if all of the cars were the same price, how much did each car cost?

43. 16 baseball gloves of the same price cost a total of $384. How much did each cost?

44. Six bowlers had a combined score of 1248. You know that they all bowled the same score. What did each Bowler Bowl?

45. 25 restaurants in Boxville contributed $625 to the restaurant association. Assuming they all contributed the same amount, how much did each contribute?

46. Mary Ann bought enough material for 6 bridesmaids dresses. Her total bill came to $138. What price would she ha ve to charge for each dress?

1.8 Divisibility Tests

Divisibility: A whole number is *divisible* by another whole number if the remainder is 0.

There are some quick test you can use to decide whether one number is divisible by another.

Divisibility by 2: A number is divisible by **2** if the number ends in 0, 2, 4, 6, or 8

Example 1:
Are all the following numbers divisible by 2?

(a) 986 is divisible by **2** because the number 986 ends in 6.

(b) 3255 is not divisible by 2 because 3255 does *not* end in 0, 2, 4, 6, or 8.

Divisibility by 3: A number is divisible by 3 if the sum of its digits is divisible by 3.

Example 2: Are all the following numbers divisible by 3?

(a) 4251

Add the digits: $4 + 2 + 5 + 1 = 12$ Since 12 is divisible by 3, the number 4251 is divisible by 3.

(b) 29,806

Add the digits: $2 + 9 + 8 + 0 + 6 = 25$ Since 25 is *not* divisible by 3, the number 29,806 is also *not* divisible by 3.

CAUTION
Be careful when testing for divisibility by *adding the digits.* This method works only when testing for divisibility by 3 or by 9.

Divisibility by 5 and by 10.
A number is divisible by **5** if it ends in 0 or 5 A number is divisible by **10** if it ends in 0

Example 3: Are the following numbers divisible by 5?

(a) 12,900 ends in 0, so it is divisible by 5.
(b) 4325 ends in 5, so it is divisible by 5
(c) 392 ends in 2, so it isn't divisible by 5.

Example 4: Are the following numbers divisible by 10?

(a) 700 and 9140 end in 0, so both numbers are divisible by 10.

(b) 355 and 18,743 do *not* end in 0, so these numbers aren't divisible by 10.

Test for Divisibility

A number is divisible by 2 if it ends in 0, 2, 4, 6, or 8.

A number is divisible by 3 if the sum of its digits is divisible by 3.

A number is divisible by 4 if the last two digits make a number that is divisible by 4

A number is divisible by 5 if it ends in 0 or 5.

A number is divisible by 6 if it is divisible by both 2 and 3.

A number is divisible by 8 if the last three digits make a number that is divisible by 8.

A number is divisible by 9 if the sum of its digits is divisible by 9.

A number is divisible by 10 if it ends in 0.

1.8 Divisibility Tests Homework

Determine if the following 28 numbers are divisible by 2, 3, 5, or 10. Put a ✓ mark in the blank if the number is divisible by the number at the top of each column. Use the divisibility tests and explain your answers. Number 1 is done for you below.

		2	3	5	10			2	3	5	10
1.	30	✓	✓	✓	✓	15.	8,302				
2.	184					16.	32,472				
3.	445					17.	612				
4.	903					18.	315				
5.	5,166					19.	2,714				
6.	21,763					20.	36,000				
7.	25					21.	836				
8.	192					22.	7,545				
9.	897					23.	242,913				
10.	500					24.	102,484				
11.	160					25.	290				
12.	635					26.	218				
13.	3,381					27.	2,020				
14.	108,605					28.	11,670				

1. 30 is divisible by 2, it ends in 0; it's divisible by 3 because 3+0 is divisible by 3; 30 is divisible by 5 because it ends in 0; 30 is divisible by 10 because it ends in 0.

1.9 Checking Division

$\begin{array}{r} quotient + remainder \\ divisor\overline{)\ dividend} \end{array}$ $\quad \begin{array}{r} 4R5 \\ 6\overline{)\ 29} \end{array}$ $\quad 6x4 = 24 \quad 24+5 = 29$ ✓

Multiply the divisor by the quotient and then add the remainder. The result is the dividend.

Example 1: Check each quotient. If the quotient is wrong, re-do the division correctly.

(a) $\begin{array}{r} 91R3 \\ 5\overline{)\ 458} \end{array}$ (b) $\begin{array}{r} 29R4 \\ 6\overline{)\ 1258} \end{array}$

1.9 Checking Division Homework

Divide and Check

1. $2\overline{)225}$ 2. $3\overline{)275}$ 3. $4\overline{)538}$ 4. $\frac{819}{5}$ 5. $5\overline{)937}$ 6. $\frac{675}{7}$ 7. $3\overline{)1885}$ 8. $8\overline{)1135}$

Check each division. If a quotient is incorrect, find the correct quotient.

9. $\begin{array}{r} 38R1 \\ 3\overline{)\ 115} \end{array}$ 10. $\begin{array}{r} 92R2 \\ 8\overline{)\ 743} \end{array}$ 11. $\begin{array}{r} 328 \\ 4\overline{)1312} \end{array}$ 12. $\begin{array}{r} 46R3 \\ 5\overline{)\ 2033} \end{array}$

#13-22 Divide & rewrite each problem using two other division symbols: ÷ or $\overline{)}$ or fraction bar.

13. $\frac{12}{12}$ 14. $\frac{9}{0}$ 15. $24 \div 0$ 16. $4 \div 4$ 17. $\frac{0}{4}$

18. $0 \div 8$ 19. $0 \div 12$ 20. $\frac{0}{7}$ 21. $0\overline{)21}$ 22. $2 \div 0$

#23-26 Divide and identify the dividend, the divisor, and the quotient.

23. $4\overline{)108}$ 24. $135/5$ 25. $324 \div 9$ 26. $8\overline{)176}$ 27. $6\overline{)9137}$

28. $9\overline{)8371}$ 29. $6\overline{)1854}$ 30. $8\overline{)856}$ 31. $4024 \div 4$ 32. $16,024 \div 8$

33. $15018 \div 3$ 34. $32008 \div 8$ 35. $\frac{26,684}{4}$ 36. $\frac{16,398}{9}$ 37. $\frac{74,751}{6}$

38. $\frac{72,543}{5}$ 39. $\frac{71,776}{7}$ 40. $\frac{77,621}{3}$ 41. $\frac{128,645}{7}$ 42. $\frac{172,255}{4}$

Check each quotient. If a quotient is incorrect, divide again to find the correct quotient.

43. $\begin{array}{r} 67R2 \\ 7\overline{)\ 4692} \end{array}$ 44. $\begin{array}{r} 663R5 \\ 9\overline{)\ 5974} \end{array}$ 45. $\begin{array}{r} 3568R2 \\ 6\overline{)\ 21,409} \end{array}$ 46. $\begin{array}{r} 25,879 \\ 4\overline{)103,516} \end{array}$

47. $\begin{array}{r} 3,003R5 \\ 6\overline{)\ 18,023} \end{array}$ 48. $\begin{array}{r} 4,208 \\ 8\overline{)33,664} \end{array}$ 49. $\begin{array}{r} 11,523R2 \\ 6\overline{)\ 69,140} \end{array}$ 50. $\begin{array}{r} 27,532R1 \\ 3\overline{)\ 82,598} \end{array}$

1.10 Exponents

The exponent tells us how many times to repeat multiplication. → 2^3 is $2 \cdot 2 \cdot 2$

$2^{15} = \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2}_{15 \text{ times}}$ On the calculator use $2 \wedge 15$

2 *is called the base*

15 *is called the exponent or power*

3^2 ← exponent, base ↗ → **3** to the second power → 3(3) → 9

2^4 is read as 2 to the fourth power which means 2(2)(2)(2) this is 16.

Examples: Simplify the exponential expression.

6^3	$\left(\frac{3}{5}\right)^2$	$\left(\frac{2}{3}\right)^5$
$6 \cdot 6 \cdot 6$	$\left(\frac{3}{5}\right)\left(\frac{3}{5}\right)$	
216	$\frac{9}{25}$	

(4^5) $(2)^4$ 3^6

4^5

Examples of more complicated exponential expressions: Notice the characteristics.
There are two exponential expressions. They are being multiplied. They have the same base.

$6^2 \cdot 6^5$ ↙	$3^4 \cdot 3^2$
$6 \cdot 6 \cdot \ 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$	
6^7	
279,936	

$(3^2)^4$ $\frac{3^7}{3^5}$ $\frac{2^5}{7^2}$

$(3^2)(3^2)(3^2)(3^2)$

$(3\cdot3)(3\cdot3)(3\cdot3)(3\cdot3)$

$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

3^8

6,651

$a^0 = 1$, *note*: $a \neq 0$ **Something raised to the zero power is *usually* "1"**

Examples: Simplify the expression.

5^0 is 1 $(17)^0$ is 4^0

51^0 $2^0 + 3^0$ $\left(\frac{3}{8}\right)^0$

1.10 Exponents *Homework*

Simplify the expression.

1. 2^3 2. $(2^3)^2$ 3. $(5^2)^3$ 4. $\frac{5^4}{5^2}$

5. $\left(\frac{2}{3}\right)^4$ 6. $\left(\frac{2}{7}\right)^2$ 7. $3^5 \cdot 3^2$ 8. $\left(\frac{2}{7}\right)^2 \cdot 2^3$

9. 4^3 10. $(2^3)(2^4)$ 11. $\frac{6^8}{6^6}$ 12. $(3^0)(5^0)$

13. $\left(\frac{4}{5}\right)^0$ 14. $\left(\frac{7^6}{7^5}\right)^2$ 15. $\left(\frac{3}{9}\right)^2 \cdot 3^4$ 16. $(3^2)(3)^4(8^0)$

1.11 Order of Operations

The four basic operations are _____, _____, _____, & _____
If an expression has all four operations, in what order should the operations be performed?

Example: Simplify $4+18\div 2-1\cdot 5+2$

Addition: $a+b \Rightarrow$ basic counting concept

Subtraction: $a-b=a+(-b) \Rightarrow$ the opposite of addition

Multiplication: → 3(2) is 2 + 2 + 2 ⇒ repeated addition

$9\cdot 4$ is $\underbrace{4+4+\ldots+4}_{9\ 4\text{'s being added}}$

Division: $a\div b=\frac{a}{b}=a\cdot\frac{1}{b} \Rightarrow$ the inverse of multiplication

Exponents (Powers): $4^7=\underbrace{4\cdot 4\cdot\ldots\cdot 4}_{7\ \text{times}} \Rightarrow$ repeated multiplication of the base

---

Example 1 **"*Multiplication is* repeated addition"**

$3\cdot 4 \Rightarrow 4+4+4 \Rightarrow 12$

$5\cdot(-2) \Rightarrow (-2)+(-2)+(-2)+(-2)+(-2) \Rightarrow -10$

$4\cdot 6 \Rightarrow$

$2\cdot(-7) \Rightarrow$

Example 2 **"*Exponents indicate* repeated multiplication"**

$3^4 \Rightarrow 3\cdot 3\cdot 3\cdot 3 \Rightarrow 81$

$(5)^3 \Rightarrow (5)\cdot(5)\cdot(5) \Rightarrow 125$

$(7)^4 \Rightarrow$

$2^{10} \Rightarrow$

Example 3 **"Exponents indicate repeated _____".**

5^2	*exponent* (power)	3^2
$5\cdot 5$	*multiplication*	
$5+5+5+5+5$	*addition*	
25	*calculation*	

Example 4 **Follow example 3.** Notice the order of operations in the example.

4^3	*exponent*
	multiplication
	addition
	final value

Example 5 **Simplify the following expressions.**

$4 \cdot 3^2 + 5 \cdot 7$ $3^2 + 4^2$ $4 \cdot 5^2 - 3 \cdot 2^4$

$2 + 4 \cdot 3^2 + 5 \cdot 7$ $4 + 3^2 + 4^2$

What if we *want* to add before multiplying? Or multiply before the exponent?
Answer: *Put the part you want to do first in parentheses.*

Grouping is indicated with parentheses ( ), with brackets [ ], and with braces { }.
If an expression has all four operations and also has exponents and grouping symbols, in what order should the 6 operations be performed?

Example 6 **Simplify the following four expressions**

$(2+3)^2$ $5 \cdot (3 - 2^3)$ $3 \cdot (5-3)^2$ $1 + 2(3^4 - 16) + 15 - 24 \div 3 \cdot 6 + 2$

Grouping is also indicated by the fraction bar. For example, to simplify the expression

$\frac{13+5-7+1}{8-3-1} + 9$ you would first need to simplify the *group* of numbers in the numerator and the *group* of numbers in the denominator.

Steps for Order of Operations

1. Grouping: Parentheses ( ) Brackets [ ] Braces { } numerator and denominator
2. Exponents (Powers)
3. Multiply or Divide in order of occurrence from left to right
4. Addition ~~or Subtraction~~

Example 7

$\frac{3^2+4^2}{7-2}$ $12(1+3)\div 6+8-2$ $8(1+3)\div 2+(3+4)\div 7-8$

1.11 Order of Operations Homework

Simplify the following expressions.

1. $3\cdot 4+5\cdot 2$
2. $5\cdot 3^2-2\cdot 7$
3. $4(3+2)^2$
4. $(5-3)^2\cdot(2+1)^2$
5. $5\cdot(3+7)-2\cdot 6$
6. $(3\cdot 4-5)^3$
7. $(14-6)^2+(5-3)^3$
8. $5\cdot(12-8)+3\cdot(2+4)$
9. $5+3\cdot 2^4$
10. $(3-1)^4+(1+2)^5$
11. $\frac{12^2+5^2}{15-2}$
12. $\frac{6^2-5^2}{22}$
13. $8+4\cdot 2-3\cdot 4$
14. $\frac{3\cdot(5^2-4^2)}{(5-2)^2}$
15. $4\cdot 3^5-3\cdot 2^7$
16. $3+2\cdot 15$
17. $5\cdot(13-7)^2+2\cdot(5-2)^2$
18. $\frac{2^3}{5}+\frac{4}{7}$
19. $\frac{3\cdot(2+3)^2}{4\cdot 5}$
20. $(13-5)^4\cdot(2+3)^2$
21. $23-2\cdot(14-9)$
22. $5^2+2\cdot 5-8$
23. $-3^4\cdot 5+(5)^2\cdot 3$
24. $5\cdot 4-3^2$
25. $2\cdot(3+4)^3$
26. $(3)^4+(5)^2$
27. $-3^4+(5)^2$
28. $(3)^4-5^2$
29. $(3\cdot 8+5^2)-(3^2-4\cdot 2)$
30. $\frac{8\cdot 3-5\cdot 2}{5^2-3^2}$

1.12 Mean, Median, and Mode are three statistical measures.

*The **mean** is the **average*** and is found by first adding all values to get the total then dividing your answer by the number of values. $\textit{mean} = \dfrac{\text{add all numbers}}{\text{number of values}}$

Example 1: Find the mean (or average) of these scores: 84, 90, 95, 98 and 88

The mean $= \dfrac{84 + 90 + 95 + 98 + 88}{5} = \dfrac{455}{5} = 91$

The mean score is 91.

Example 2: The sales of photo albums at Sarah's Card Shop for each day last week were \$86, \$149, \$103, \$118, \$117, \$126, \$158 and \$149. Find the mean sales.

Answer: The mean sales amount is _____.

*The **median** is the middle number* in a list of numbers when listed **in order** from **smallest** to **largest**.

Steps to *finding the Median of a set of numbers*

1. Arrange the numbers from smallest to largest.
2. If there is an odd number of values, the middle value is the median.
   If there is an even number of values, the average of the two middle values is the median.

Example 1: Find the median of these scores: 84, 90, 95, 98 and 88

Answer: The median score is _____.

Example 2: The sales of photo albums at Juanita's Card Shop for each day last week were \$86, \$103, \$149, \$118, \$117, \$126, \$158 and \$149. Find the median for the sales of photo albums.

Answer: The median score is _____.

29. A car travels 220 miles in 4 hours. What's the rate of the car in miles per hour?

30. A train travels 360 miles in 5 hours. What's the rate of the train in miles per hour?

31. The flow of water from a water faucet can fill a 3-gallon container in 15 seconds. Give the ratio of gallons to seconds as a rate in gallons per second.

32. A car travels 95 miles on 5 gallons of gas. Give the ratio of miles to gallons as a rate in miles per gallon.

Twenty-one people work in an office which has 15 desks and nine computers.

33. What is the ratio of desks to people?

34. What is the ratio of computers to people?

35. The ratio of females to males is 6 to 7. The class has 39 students. How many females are there?

36. The ratio of females to males is 6 to 7. The class has 39 students. How many males are there?

Be the change you want to see in the world.

Mahatma Gandhi

3 GEOMETRY

3.1 Points, Lines, Planes, & Angles

Draw each of the following representations.

Point Dot, no length, width or thickness

Line Goes through two points, extends infinitely in both directions (A line is an infinite number of points.)

Line AB Half Line AB **Ray** AB **Line Segment** AB

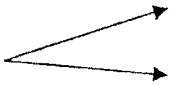
Plane A flat surface with no thickness & no boundaries

A ray is a set of points starting at a point and continuing in one direction forever.

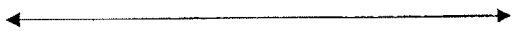
Note: A ray can rotate 360° or more

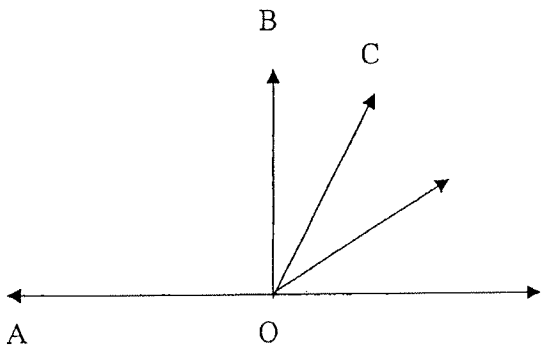
Angle Formed by two rays starting at the same point

$\measuredangle ABC = \measuredangle CBA$

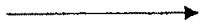


Acute angles are between 0 degrees and 90 degrees. They are sharp (i.e. pointy) angles.
A right angle measures 90 degrees. The rays are perpendicular $\perp$.
Obtuse angles are between 90 degrees and 180 degrees. They are dull angles.
Straight angles measure 180 degrees. They are lines.
Reflexive angles are between 180 degrees and 360 degrees.





Example 1. Draw and describe a 30 degree angle.



Answer: A 30 degree angle is 1/6 of a straight angle. It is an acute angle.

Example 2. Draw and describe a 120 degree angle.

Example 3. Draw and describe each of the following angles.

zero degrees	60 degrees	90 degrees right angle	180 degrees straight angle	360 degrees reflexive angle

Similar Figures are figures that have the same shape but not necessarily the same size.
Draw two similar rectangles with corresponding sides and corresponding angles.

Complementary Angles are two angles whose measures add up to 90°.
Draw two complementary angles.

Supplementary Angles are two angles whose measures add up to 180°.
Draw two supplementary angles.

Parallel Lines are lines that have the same slope and never intersect.
Draw two parallel lines.

Intersecting Lines are lines that intersect at one point.
Draw two intersecting lines which form 20 degrees.

Perpendicular Lines are lines that intersect at a 90° angle.
Draw two perpendicular lines.

3.1 Angles Homework Draw & describe the angle: acute, right, obtuse, straight, reflexive

1. Zero degree angle or 360 degree angle	2. 30 degree angle.	3. 45 degree angle	4. 60 degrees
5. 90 degree angle.	6. 120 degree angle.	7. 135 degrees	8. 150 degrees
9. 180 degree angle.	10. 210 degree angle.	11. 225 degree angle	12. 240 degrees
13. 270 degree angle.	14. 300 degree angle.	15. 315 degree angle	16. 330 degrees

3.2 Two-Dimensional Figures and their characteristics

Always draw the geometric figure and label the necessary features before doing any work.

Triangles 10.2 Veronica's

A Triangle is a geometric figure of 3 sides
* The sum of the measures of the 3 angles of any triangle is 180°

Types of Triangles (pictures)

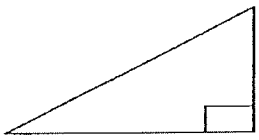
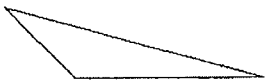
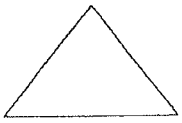
Acute	Right	Obtuse
All angles are acute	One right angle	One angle is obtuse
Isosceles	**Equilateral**	**Scalene**
Two side have equal length	all side are equal	no two sides are equal

Triangles have three sides.
The three angles of any triangle add up to 180 degrees.
The base is the horizontal segment that the triangle sits on.
The height is vertical segment from the top of the triangle to the base.
The height of a triangle is perpendicular to the base of the triangle.

1) 2 sides are the same length and 2 angles have the same number of degrees = **Isosceles**
2) Has 2 legs and a hypotenuse; one angle is 90°; $a^2 + b^2 = c^2$ **Right Triangle**
3) All sides are different lengths and all the angles have different number of degrees = **Scalene**
4) All sides are the same length and all angles are 60 degrees= **Equilateral**

Triangle shapes and names:

Isosceles Triangle *Right Triangle* *Scalene Triangle* *Equilateral Triangle*

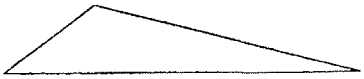
$a^2 + b^2 = c^2$

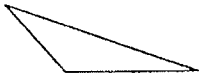
2 sides are the same length
the same length
2 angles have the same
60 degrees
number of degrees

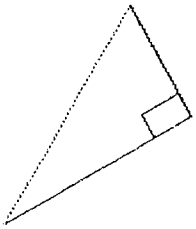
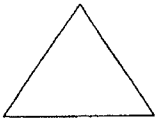
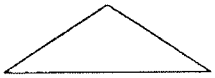
The Pythagorean Theorem
Has 2 legs and a hypotenuse
One angle is 90 degrees

All sides are different lengths
All the angles have different number
of degrees

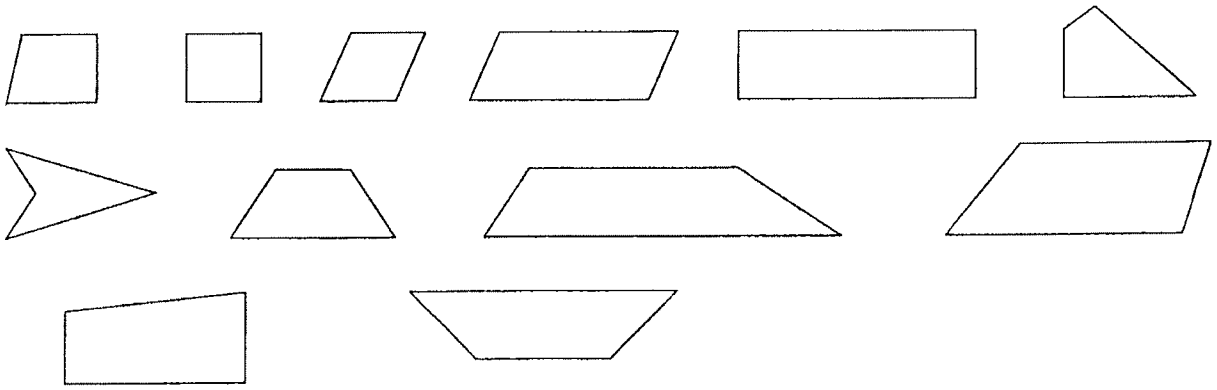
All sides are
All angles are





Quadrilateral ⇔ = **a 4-sided geometric figure.**
Parallelogram ⇒ 2 pairs of sides are parallel.
Rectangle ⇒ 4 right angles.
Rhombus ⇒ 4 equal sides.
Square ⇒ 4 equal sides **and** 4 right angles.
Trapezoid ⇒ 1 pair of parallel sides.



Quadrilaterals have _____sides and_____angles.
The four angles add up to _____ degrees
The height is perpendicular ⊥ to the base.
Quadrilaterals are made up of two triangles.

Parallelograms have 2 pairs of parallel sides. **Parallelogram** ⇒ 2 pairs of sides are parallel.

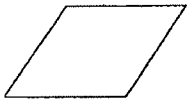
The base and the top are the same _____
The left side and the right side are the same _____
The **height** is perpendicular ⊥ to the base.
The height does not equal the length of left or right side.
Is a parallelogram a rectangle?
Answer: _____

Rectangles have four _____angles. **Rectangle** ⇒ 4 right angles.

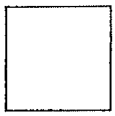
Rectangles have two pairs of_____sides.
A rectangle is made up of _____triangles.
Adjacent sides of a rectangle are _____
The base and the top are the same _____
The left side and the right side are the same _____
The base is sometimes called the _____
The side is sometimes called the height or the _____
Is a rectangle a parallelogram? Answer: _____
Is a rectangle a square? Answer: _____

Rhombus ⇔ 4 equal sides. Draw a rhombus.

A rhombus has _____equal sides. This is like a slanted square or a regular square.
Is a rhombus a square? Answer: _____



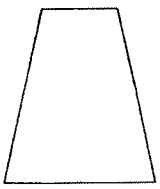
Square ⇔ 4 equal sides **and** 4 right angles.
The sum of the angles = _____

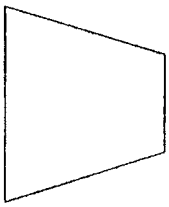


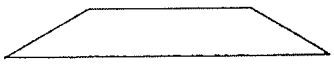
A square has four equal sides **and** four right angles. Is a square a rhombus?
Answer: _____

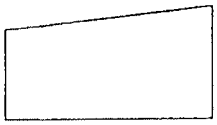
Is a square a rectangle? Answer: _____.

Trapezoids have **one** pair of parallel sides.

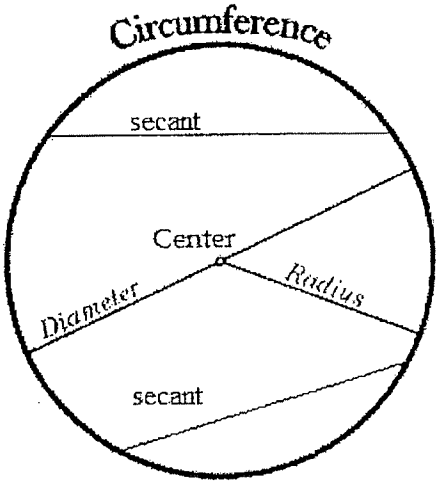








Circle



1. **A circle is made up of points** which are all the same distance from a point called the center.
2. **The diameter is a set of points** starting on the circle going through the center to the other side of the circle.
3. **The secant is the set of points** starting on the circle not through the center to the other side of the circle.
4. **The radius is the set of points** from the center to the circle.
5. **The circumference is the distance** around the circle. This is the perimeter of the circle.
6. **A circle has 360 degrees.**

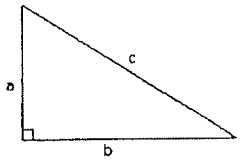
3.2 Two-Dimensional Figures Homework

Draw each of the following and describe the characteristics of each.

1. Isosceles Triangle	2. Right Triangle	3. Scalene Triangle
4. Equilateral Triangle	5. Acute angle	6. Obtuse angle
7. Reflexive angle	8. Straight angle	9. Right angle
10. Quadrilateral	11. Parallelogram	12. Rectangle
13. Rhombus	14. Square	15. Trapezoid
16. Circle	17. diameter	18. radius
19. secant	20. Circumference	

21. Draw a triangle. How many degrees are there in a triangle? Use complete sentences.
22. Draw a rectangle. How many degrees are there in a rectangle?
23. Draw a quadrilateral. How many degrees are there in a quadrilateral?
24. Draw a circle. How many degrees are there in a circle?

3.3 The Pythagorean Theorem

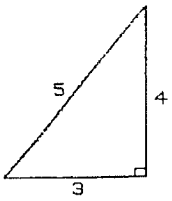


$a^2 + b^2 = c^2$

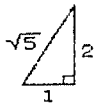
The two sides which make the right angle are called the legs and the lengths are represented by a and b. The last side is the longest side and it is called the hypotenuse. Its length is represented by the letter c.

Pythagorean Theorem: $a^2 + b^2 = c^2$ Add the squares of the legs. This equals the hypotenuse squared.

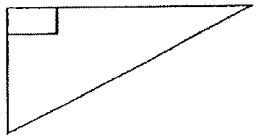
In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



$3^2 + 4^2 = 5^2$
$9 + 16 = 25$



$1^2 + 2^2 = (\sqrt{5})^2$
$1 + 4 = 5$



Example 1. Draw a right triangle where a is 6 inches and b is 8 inches. Find the length of c.

Example 2. A wire from the top of a 12-foot pole is fastened to the ground 5 feet from the bottom of the pole. What is the length of the wire?

Example 3. Draw a right triangle where a = 12 cm and c = 20 cm. Find the length of b.

Example 4. Draw a right triangle where a = 7 ft and c = 25 ft. Find the length of b.

Example 5. Draw a right triangle where b = 5 in. and a = 4 in. Find the length of c.

3.3 The Pythagorean Theorem Homework

Draw each triangle. **Find the length of the missing side.**
If the answer is not a whole number, then give the actual answer **and** the answer to two (2) decimal places.

1. a=5 yd. and b= 5 yd. 2. a=15 in., b=9 in. 3. a=6 ft, b= 6ft. 4. a=18 m, c= 30 m.

5. A wire from the top of a 24-foot pole is fastened to the ground 10 feet from the bottom of the pole. How long is the wire?

6. A ladder is leaning against the top of a 15-foot wall. If the bottom of the ladder is 20 feet from the wall, how long is the ladder?

7. a=5 ft., b= 2 ft.

8. a= 5, b = 7

9. The width of a television screen is measured by the length of the diagonal. If the diagonal is 27 inches, and the height is 17 inches, find the length of the television.

10. Is a triangle with sides of 5, 12, and 13 meters a right triangle?

11. Is a triangle with sides of 2, 2, and 3 feet a right triangle?

Find the missing side.

12. a=15, c=17. 13. b=45, c=53. 14. a=5, b=7. 15. a=1, b=7.

16. The lengths of the three sides of a triangle are 8, 15, and 17. Determine whether the triangle is a right triangle.

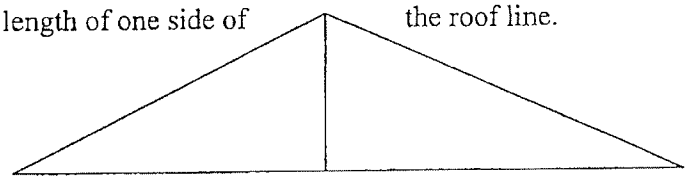
17. The lengths of the three sides of a triangle are 7, 24, and 26. Determine whether the triangle is a right triangle.

18. The lengths of the three sides of a triangle are 6, 8, and 10. Determine whether the triangle is a right triangle.

19. The lengths of the three sides of a triangle are 9, 39, and 40. Determine whether the triangle is a right triangle.

20. A tree casts a shadow 24 feet long and a man 6 feet tall casts a shadow 4 feet long. Find the height of the tree.

21. The gable end of the roof is divided in half by a vertical brace which is 8 feet tall. The base is 30 feet. Find the length of one side of the roof line.



22. Find the length of the hypotenuse of the right triangle whose legs measure 5 feet and 12 feet. Draw and label the figure.

23. A 45-foot tall ladder is fifteen feet from the base of a house. How tall is the house? Draw and label the figure.

24. Find the length of the missing leg of the right triangle when the hypotenuse is equal to 60 inches and one leg is 40 inches. Draw and label the figure.

25. To get from point A to point B, you must avoid walking in a pond. To avoid the pond, you must walk 18 yards south and 6 yards east. If you had on the proper shoes and could walk through the pond, how many yards would you have walked? Draw and label the figure.

26. Thumbelina is 10 centimeters tall. She casts a 15 centimeter shadow. What is the distance from the top of her head to the tip of her shadow? Draw and label the figure.

27. To prove a point, architect Carlos built a pole that casts a 14-foot shadow. The distance from the top of the pole to the edge of the shadow is 36 feet. How tall is the pole? Draw and label the figure.

28. A suitcase is 12 inches high and the diagonal distance across it is 48 inches. What is the width of the suitcase? Draw and label the figure.

29. A 15-foot house has a 17 foot cable line stretched from the roof. What is the distance from the base of the house to the end of the cable wire? Draw and label the figure.

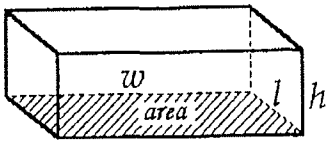
30. Olivia and Andrea start out at the same point. Olivia travels 8 yards south and Andrea travels 22 yards southeast. What is their final distance away from each other? Draw and label the figure.

31. Two sprinters run 3 feet south and 4 feet east. What is the shortest distance you have to travel to reach their starting point? Draw and label the figure.

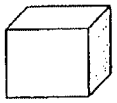
32. The Martinez family plans to carpet their new family room which is 30 feet long and 21 feet wide. The carpeting costs $13 per square yard. How many square yards will the family need and what will be the total cost for the carpeting?

3.4 Three-Dimensional Figures

A rectangular prism is called a rectangular box or a rectangular base solid.
It is called a rectangular prism because the base is a _____
It has 12 edges. Number them 1 to 12 in the diagram.

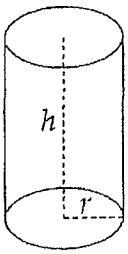


A cube has all its sides the same length.
The base is a _____
How many edges does it have?

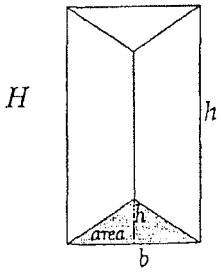


Answer: The cube has _____ edges.

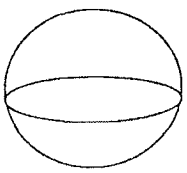
A circular prism is the shape of a can. This is also called a circular base solid or a cylinder.
It is called a circular prism because the base is a _____.
If you unroll the can, you will have two _____
and a _____



A triangular prism is the shape of a _____?_____
The base is the shape of a _____
If you laid it out flat, you would have two _____
and a _____



A sphere is the shape of a ball.
It is made up of points which are equidistant from a point called the center; it has a radius and a diameter.



3.4 Three-Dimensional Figures Homework

Label the parts ℓ for length, w for width, r for radius, d = diameter, h for height, b = base, c for center.

1. Draw two different rectangular prisms and label the parts

2. Draw two different circular prisms and label its parts.

3. Draw two different triangular prisms and label its parts.

4. Draw a sphere and label its parts.

Write a complete sentence to answer the following questions.

5. How many sides does a rectangular prism have? Draw one and number the sides.

6. How many sides does a cube have? Draw one and number the sides.

7. How many sides does a triangular prism have? Draw one and number the sides.

8. How many corners does a rectangular prism have? Draw one and number the corners.

9. How many corners does a cube have? Draw one and number the corners.

10. How many corners does a triangular prism have? Draw and number the corners.

3.5 Perimeter

The perimeter is the distance around the rim of a figure. It is sometimes considered a fence. The perimeter is represented by the capital letter P.

Triangles and perimeter

For a triangle, the perimeter is the sum of the lengths of the three sides.

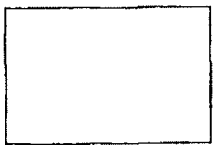
Perimeter → $P = a + b + c$ *units*

Example 1: Draw the triangle then find the perimeter of a triangle whose sides are 5", 12", and 3".

Answer: P = a + b + c Draw the picture here
P = 5 + 12 + 3
P = 20 inches

Example 2: What is the perimeter of a triangle whose sides are each 13"? Draw the triangle.

Rectangles and perimeter

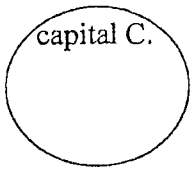


The perimeter of a rectangle is the sum of the lengths of the four sides.

P = b + b + h + h
P = 2b + 2h or *Perimeter* → $P = 2l + 2w$ *units*

Example 3: What is the perimeter of a rectangle whose base is 15' and the height is 12'?

Circles and perimeter



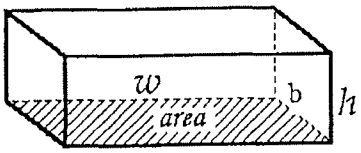
For a circle, the perimeter is called the circumference and is represented by a capital C.

π is an irrational number called "pi." It's value is approximately 3.14 or $\frac{22}{7}$.

P= C = π * d

Example 4: What is the circumference of a circle with a radius of 5m? *Give exact answer and answes rounded to one decimal place (where appropriate).*

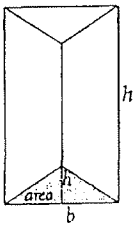
Rectangular prisms and perimeter



P = b+b+b+b+w+w+w+w+h+h+h+h
P = 4b + 4w + 4h units

Example 6: Find the perimeter of the rectangular prism where b = 6', w = 12', and h ' 12'.

Triangular prisms and perimeter

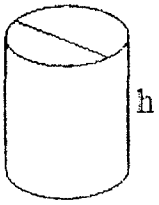


P = a + b + c + h + h + h + a + b + c
P = 2a + 2b + 2c + 3h units

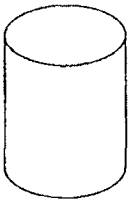
Example 5: What is the perimeter of the triangular prism whose long edges measure 13" and the short edges measure 7", 3", and 6"? (Note: Be sure to draw and label the figure properly.)

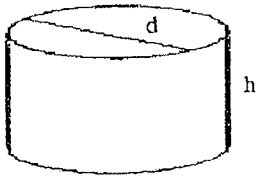
Circular Cylinder or Circular Prism

A circular prism is the shape of a can.
It is called a circular prism because the base is a circle.
If you unroll the can, you will have two circles & a rectangle.



$P = \pi d + \pi d + h + h$
$P = 2\pi d + 2h$





3.5 Perimeter Homework

Give exact answer and answer rounded to one decimal place.

1. What is the distance around a figure called?
2. Draw each of the following figures and give the formula for the perimeter (or circumference).

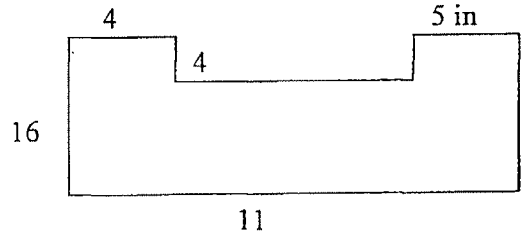
Triangle	Rectangle	Circle	Rectangular Prism
Triangular Prism	Square	Rhombus	Parallelogram

3. What is the perimeter of a triangle whose sides measure 3.1 inches, 5.175 inches, & 7.35 inches? Give the answer in two forms: as a **decimal** and as a **mixed number**.

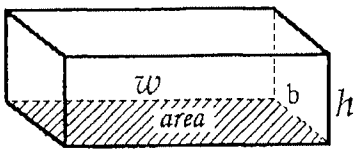
A circle has a radius of 5.2 meters. Use the decimal approximation of pi.

4. Find the diameter of the circle.
5. Find the circumference of the circle.

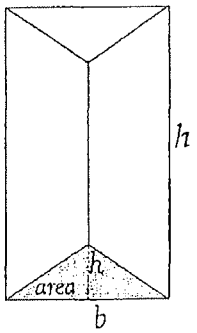
6. A triangle has sides which measure 4 1/3 yds, 8 3/8 yds, and 2 3/4 yds. The height of the triangle is 3 yds. Find the perimeter.
7. Find the perimeter of a rectangle whose sides measure 4 1/2 by 8 2/3 in.
8. Find the circumference of a circle with radius 5 feet.
9. Assume a right triangle has sides of 2 1/2", 3 1/2", and 5". What is the perimeter of the triangle?
10. Draw and label a circle with radius 12.7 feet. Give the formula for the circumference of the circle.
11. Give the symbol for "pi" and two approximate values of "pi."
12. Give the approximate value of the number "e."
13. Find the perimeter of the following figure.



14. Find perimeter of the following shape given that the length of b is 5 inches, the length of h is 3 inches, and the length of w is 17 inches.



15. Find the perimeter of the following figure if the sides of the triangle are 4 meters, 3 meters, and 2 meters, and the height of the figure is 7 meters.



16. A rectangle has a length of 17.2 feet and a width of 6.9 feet. Find the perimeter.

3.6 Area

The area is the number of squares in the interior of a figure.

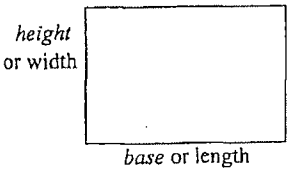
2-dimensional areas and 3-dimentional areas

Area & Circumference 10.4 Veronica's

Formulas:

- Area of a Rectangle: A = L x W (Picture)
- Area of a Square: $A = S^2$
- Area of a Parallelogram: A = bh
- Area of a Triangle: $A = \frac{1}{2}bh$
- Area of a Trapezoid: $A = \frac{1}{2}h(a+b)$
- Area of a Circle: $A = \pi r^2$ $\pi = 3.14$ diameter = r + r = 2r
- Circumfrence – Distance around the circle: $C = 2\pi r$ $A = \pi r^2$

Rectangles and area



Example 1:
Consider a rectangle which has a base of 4 inches and a height of 3 inches.

How many squares are in the interior? Show the squares in the rectangle.
What is the area of the rectangle?

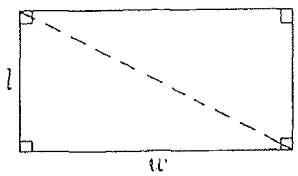
Answer: A = _____ square inches

There are_____ squares in the interior of the rectangle.
The area of the rectangle is _____ square feet (i.e. _____ feet squared or _____ ft^2).

To find the area of a rectangle multiply the base times the height.

A = b * h or A = bh or A = lw

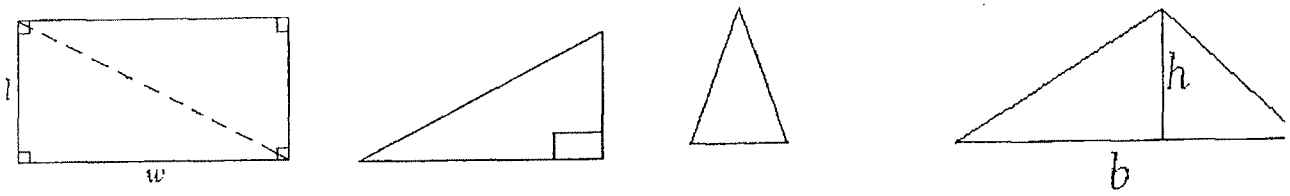
Area $\rightarrow A = lw$ square units



Triangles and area

All triangles are half of a rectangle.

$Area \rightarrow A = \frac{1}{2}bh = \frac{bh}{2}$ square units

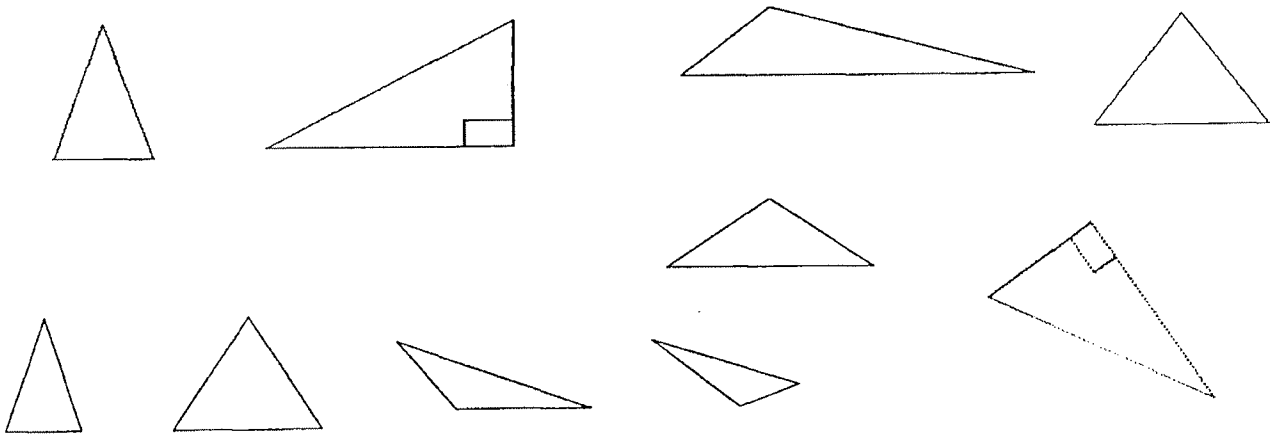


Example 2:
Consider a triangle which has a base of 4 inches and a height of 3 inches.
How many squares are in the interior? Show the squares in the triangle. This is not easy to do.
What is the area of the triangle?

Answer: A =
There are ___ squares in the interior of the rectangle.
The area of the rectangle is ___ square ft or ____ft squared.

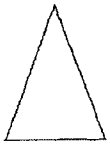
To find the area of a triangle you can multiply the base times the height and divide by 2.

$A = \frac{1}{2}b * h$ or $A = \frac{1}{2}bh$ or $A = \frac{1}{2}bh$ or $A = \frac{1}{2}lw$ or $A = \frac{bh}{2}$ or $A = \frac{lw}{2}$ units^2

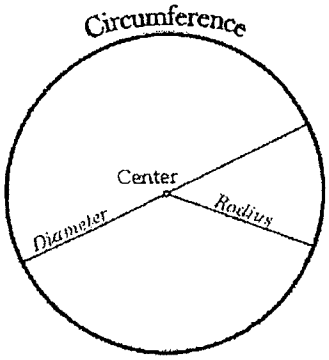


Redraw each triangle so that one of the sides is horizontal. This is called the base.
Put your pencil on the highest point on the triangle. Draw a line segment from this point to the base.
Make sure the line segment is perpendicular $\perp$ to the base. This is the height of the triangle.
Label the base and height of each triangle by writing "b" or "h" to identify the base and height.

Example 1: Find the area of the triangle whose sides measure 7', 7', and 2'. The height of the triangle is 6'.

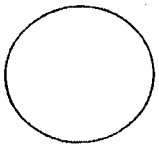


Circles and area



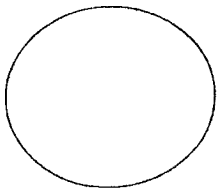
$Area \rightarrow \quad A = \pi \cdot r^2$ units^2

The area of a circle is π times the radius squared.
Give exact answer and answer rounded to one decimal place (where appropriate).



Example 3:
Consider a circle with a radius of 5 cm.
How many squares are in the interior? Show the squares in the circle. This is not easy to do.
What is the area of the circle?
Do this problem twice: once using 3.14 for pi and the second time using $\frac{22}{7}$.

Answer: A =



Example 4:
Consider a circle with a diameter of 14 inches.
How many squares are in the interior? Show the squares in the circle. This is not easy to do.
What is the area of the circle?
Do this problem twice: once using 3.14 for pi and the second time using $\frac{22}{7}$.

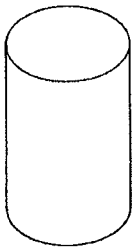
Answer: A =

3-Dimensional Areas

Area of a circular prism *Give exact answer and answer rounded to one decimal place (where appropriate).*

Example 5:

Find the area of the circular prism whose radius is 3 inches and the height is 7 inches.



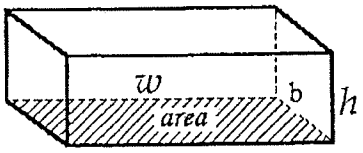
$$A = \pi r^2 + \pi r^2 + d\pi h$$

$$Area \rightarrow \quad A = 2\pi r^2 + dh\pi \text{ units}^2$$

Area of a rectangular prism

Example 6:

Find the area of the rectangular prism whose base is 3 feet, width is 4 feet, and height is 2 feet.



$$A = bh_{\text{right end}} + bh_{\text{left end}} + hw_{\text{front}} + hw_{\text{back}} + bw_{\text{bottom}} + bw_{\text{top}}$$

$$A = 2bh + 2hw + 2bw \text{ units}^2$$

3.6 Area Homework

Give exact answer and answer rounded to one decimal place.

1. Draw each figure and give the formula for the area of each.

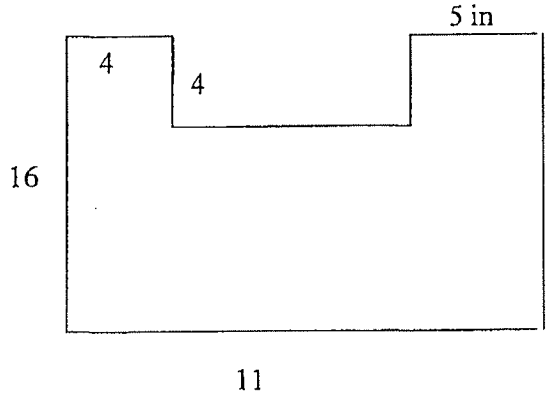
Triangle	Rectangle	Circle	Rectangular Prism
Triangular Prism	Square	Rhombus	Parallelogram

2. What is the area of a rectangle whose base is 8 2/5 inches and its height is three times this number.? Give the answer in two forms: as a **decimal** and as a **mixed number**.

3. A circle has a radius of 5.2 meters. Use the decimal approximation of pi. Find the area of the circle.

4. A triangle has sides which measure 4 1/3 yds, 8 3/8 yds, and 2 3/4 yds. The height of the triangle is 3 yds. Find the area.

5. Assume a right triangle has sides of length 2 1/2", 3 1/2", 5". What's its area?
6. Find the area of the following shape.



7. What is the area of a rectangle whose base is 8 2/5 inches and its height is three times this number? Give the answer in two forms: as a **decimal** and as a **mixed number.**

8. A triangle has sides which measure 4 1/3 yds, 8 3/8 yds, and 2 3/4 yds. The height of the triangle is 3 yds. Find the area of the triangle.

9. Find the area of a rectangle whose sides measure 4 1/2 by 8 2/3 in.

10. Find the area of a circle with a diameter of 10 feet.

11. Assume a right triangle has sides of 2 1/2", 3 1/2", and 5". What is the area of the triangle?

12. Draw and label a circle with diameter 25.4 feet, then give the formula for the area of the circle. Find the area.

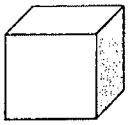
13. A rectangle has a length of 17.2 feet and a width of 6.9 feet. Find the area.

14. A circle has a radius of 7.5 inches. What is the diameter? Find the circumference and the area.

Describe the purpose of the following formulas. Draw and label the appropriate geometric figure.

15. $P = 2b + 2h$
16. $P = a + b + c$
17. C = pi times d
18. A = b times h
19. A = 1/2 b times h
20. $A = \text{pi times } r^2$

3.7 Volume



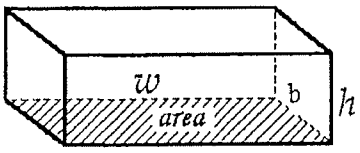
A cube is a 3 dimensional figure whose sides are all the same size.

Volume tells how many cubes are in the interior of a 3-dimensional figure.

Volume 10.5 Veronica's

Rectangular Prism: $V = L \times W \times H$
Right Circular Cylinder: $V = \pi r^2 h$
Right Pyramid: $V = \frac{1}{3}Bh$, B = area of base
Right Circular Cone: $V = \frac{1}{3}\pi r^2 h$
Sphere: $V = \frac{4}{3}\pi r^3$

To find the volume of a box, multiply the area of the base times the height of the box.

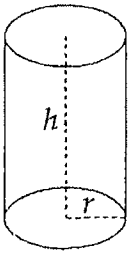


$V = A \cdot h$
$V = l \cdot w \cdot h$ or $V = b \cdot h \cdot w$ cubic units

Example 1:
Consider a box with a base of 3 inches, height of 2 inches, and a width of 14 inches.
How many cubes are in the interior? Show the cubes in the box. This is not easy to do.
What is the volume of the box?

Answer: A =

The volume of a cylinder equals the area of the base times the height of the cylinder.



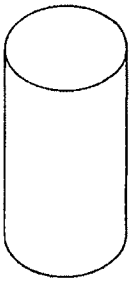
Give exact answer and answer rounded to one decimal place (where appropriate).

Volume

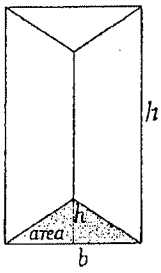
$V = A \cdot h$

$V = \pi r^2 \cdot h$ cubic units *or* units^3

Example 2: Find the volume of the cylinder whose radius is 5' and height is 12'.



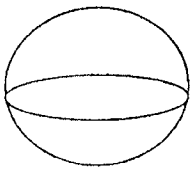
To find the volume of a triangular prism, multiply the area of the base times the height.



Find the volume. b = 6m, h = 4m, H = 10m

Example 3: Find the volume of a triangular prism which is 20 inches tall, the b is 12 inches, and the height of the base is 5 inches.

To find the volume of a sphere multiply 4/3 times πr^2 . *Give exact answers and answers rounded to one decimal place.*



$V = \frac{4}{3}\pi r^3$ cubic units *or* units^3 Area = $4\pi r^2$

Example 4: Find the volume of the sphere with a radius of 6 yards.

Example 5: Find the volume of the sphere with a diameter of 10 cm.

3.7 Volume Homework Draw and label each geometric figure. *Give exact answers* and also *answers rounded to one decimal place.*

1. How many cubic feet of water are needed to fill a spherical water tank with a radius of 15 feet?
2. Find the volume of the circular cylinder with diameter 6 cm and height 10cm.
3. Find the volume of a rectangular solid with dimensions of 3 by 4 by 5 centimeters.
4. Find the volume of a rectangular solid with dimensions of 5 by 8 by 10 meters.
5. Find the volume of a prism whose base is a right triangle with legs 3 and 4 meters long and whose height is 8 meters.
6. Find the volume of a prism whose base is a right triangle with legs 5 and 12 feet long and whose height is 10 feet.
7. Find the volume of a sphere with a radius of 9 inches.
8. Find the volume of a sphere with a diameter of 10 feet.
9. Find the volume of a cylinder with a height of 12 meters and a circular base with a radius of 6 meters.
10. Find the volume of a cylinder with a height of 4 meters and a circular base with a diameter of 18 meters.
11. A classroom is 40 feet long, 30 feet wide, and 9 feet high. Find the number of cubic feet of air in the room.
12. How many cubic feet are there in a cubic yard?
13. How many cubic inches are there in a cubic foot?
14. The largest refrigerator in New Mexico has a capacity of 25.2 cubic feet. How many cubic inches is this?
15. The lifting power of a spherical balloon depends on its volume. How many cubic feet of gas will a balloon hold if it is 40 feet in diameter?
16. A box of cereal measures 3 by 8 by 10 inches. The manufacturer plans to market a smaller box that measures $2\frac{1}{2}$ by 7 by 8 inches. By how much will the volume be reduced?
17. What units are used to measure area and what units are used to measure volume?

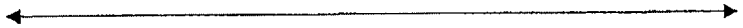
I would hope that a wise Latina woman with the richness of her experiences would, more often than not, reach a better conclusion than a white male who hasn't lived that life.

A wise role model: Sonia Sotomayor becomes first Latina Supreme Court Justice, 2009

Chapter 4 – Integers (i.e. Signed Numbers)

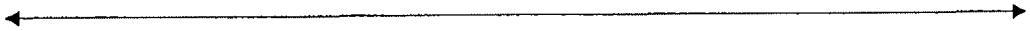
4.1 Number Line and Concepts

Review the Basic Operations Adding and Subtracting sections 1.4 and 1.5
Number line: Graphical representation of the numbers. Mark the location of zero.



Mark the location of zero then plot the following numbers on the number line.

$-2, 3, \frac{1}{2}$, *and* $-\frac{5}{3}$



No homework for 4.1

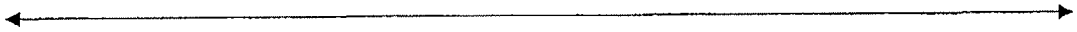
4.2 Adding ~~(and subtracting)~~ Signed Numbers

Draw the number line. Mark the location of zero.

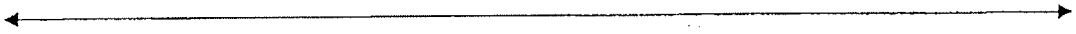
1. Start at the zero.
2. Go in the direction of the first number.
3. Travel in the direction of the second number.
4. The answer is the number where you ended up.

Example 1 $3+5$

Example 2 $-3+7$

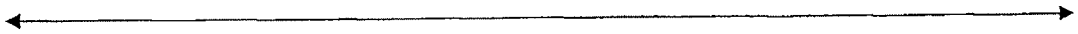


Example 3 $-8+5$



Example 4 $5+(-9)$ plus times a minus is a minus

$5 - 9$



Example 5 $(-2)+(-3)$ plus times a minus is a minus

Example 6 **Use the number line to simplify the 6 expressions given below.**

$-12+(-15)$ $17+(-9)$ $5+(-12)$

$8+9$ $(-24)+17$ $(-25)+(-33)$

Subtracting Signed Numbers

Subtracting is adding the opposite.

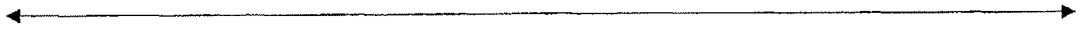
$a-b=a+(-b)$

The opposite of a negative is a positive

$-(-a)=+a$ *or* $-(-a)=a$

OR

1. A plus times a minus is a **minus.** (different signs produce a minus)
2. Every other multiplication is a **plus.** i.e. *a minus times a minus is a plus*
   a plus times a plus is a plus



The Opposite of a Negative is a positive.

Example 7 **Simplify the 6 expressions given below.**

$5-8$ $5+(-8)$ $-4-(-3)$ $-4+3$ $-5-6$ $-5+(-6)$

Example 8 **Simplify the 5 expressions given below.**

$4-12$ $-5-7$ $(-4)-(-7)$ $18-13$ $(-3)-5$

Example 9 **Simplify the 5 expressions given below.**

$-5-9$ $14-28$ $-13+8$ $16+24$ $-25-(-34)$

4.2 Adding ~~(and subtracting)~~ Signed Numbers Homework

For problems 1-6, show the addition on the number line.

1. $3+4$ 2. $5+(-3)$ 3. $(-2)+(-4)$
4. $(-7)+5$ 5. $5+(-8)$ 6. $(-3)+(-8)$

Perform the indicated operation to simplify the following expressions.

7. $8+(-14)$ 8. $(-9)+(-8)$ 9. $-15+6$
10. $15+23$ 11. $15+(-23)$ 12. $(-15)+(-23)$
13. $24-(-25)$ 14. $-12-13$ 15. $-12+13$
16. $(-8)-(-3)$ 17. $14-(-7)$ 18. $-23+(-12)$
19. $45+(-34)$ 20. $(-24)-(-22)$ 21. $15+(-23)$
22. $45-25$ 23. $-45-25$ 24. $(-27)-(-22)$
25. $-52-25$ 26. $-52+25$ 27. $52-(-25)$
28. $(-24)+(-16)$ 29. $(-24)-(-16)$ 30. $18-(-9)$
31. $-24-36$ 32. $-24-(-36)$ 33. $25+(-36)$
34. $54-(-18)$ 35. $-54+18$ 36. $-76-85$

4.3 Multiplying and Dividing Signed Numbers

Multiplying $\Rightarrow$ repeated addition $\Rightarrow 3\cdot 8 \Rightarrow \underbrace{3+3+\ldots+3}_{8 \text{ times}}$

Example 1 Multiply the numbers below.

$2\cdot 5 \Rightarrow 5+5 \Rightarrow 10$

$5\cdot 2 \Rightarrow 2+2+2+2+2 \Rightarrow 10$

$4\cdot 7 \Rightarrow 7+7+7+7 \Rightarrow 28$

$6\cdot 3 \Rightarrow$

$3\cdot 8 \Rightarrow$

Now considering signed numbers. Note: $-(-a)=a$

$3\cdot(-5) \Rightarrow -5-5-5 \Rightarrow -15$

$4\cdot(-2) \Rightarrow -2-2-2-2 \Rightarrow -8$

$-3\cdot 4 \Rightarrow$

$(-2)\cdot(-3) \Rightarrow$

$-5\cdot(-4) \Rightarrow$

$-6\cdot 5 \Rightarrow$

Notice a pattern.

$pos \cdot pos \Rightarrow pos$

$neg \cdot neg \Rightarrow pos$

$pos \cdot neg \Rightarrow neg$

$neg \cdot pos \Rightarrow neg$

OR if the signs are different $\Rightarrow neg$ all other multiplication is positive

---

Multiplying signed numbers.

1. Multiply the numbers together.
2. If the signs are different $\Rightarrow$ *neg answer* *otherwise* the answer is positive

---

Example 2 Multiply the signed numbers below.

$5\cdot(-8)$ $(-3)\cdot(-7)$ $-8\cdot 4$ $12\cdot 5$ $15(-8)$ $(-9)(3)$

Dividing signed numbers is like multiplication.

If the signs are different $\Rightarrow$ *neg answer* *otherwise* the answer is positive

---

$\frac{a}{b}=c \Rightarrow b\cdot c=a$

Example 3 Divide the signed numbers below.

$\frac{12}{4}=3 \Rightarrow 4\cdot 3=12$ $\qquad$ $\frac{30}{6}=\_\_\_ \Rightarrow$ _____

So, let us consider the following cases. What is the sign of the c - value?

$\frac{pos}{pos}=c \Rightarrow pos\cdot c=pos \Rightarrow c=$ _____

$\frac{neg}{neg}=c \Rightarrow neg\cdot c=neg \Rightarrow c=$ _____

$\frac{pos}{neg}=c \Rightarrow neg\cdot c=pos \Rightarrow c=$ _____

$\frac{neg}{pos}=c \Rightarrow pos\cdot c=neg \Rightarrow c=$ _____

Example 4 Divide the signed numbers below.

$\frac{18}{3}$ $\quad$ $\frac{-16}{5}$ $\quad$ $36\div(-9)$ $\quad$ $\frac{52}{-13}$ $\quad$ $(-18)\div 6$ $\quad$ $\frac{105}{-7}$ $\quad$ $\frac{-24}{4}$ $\quad$ $\frac{-42}{-6}$

4.3 Multiplying and Dividing Signed Numbers Homework

Multiply or Divide as indicated.

1.	$5\cdot 4$	2.	$(-3)\cdot 6$	3.	$(-5)\cdot(-3)$	4.	$4\cdot(-7)$
5.	$7(-5)$	6.	$-3\cdot 8$	7.	$23\cdot(-24)$	8.	$(-12)(-20)$
9.	$(-15)\cdot 4$	10.	$(-8)(-7)$	11.	$(-8)\cdot 7$	12.	$(-14)(-15)$
13.	$(-24)\div 4$	14.	$35\div(-7)$	15.	$(-45)\div(-5)$	16.	$-28/4$
17.	$-60/-5$	18.	$93/-3$	19.	$54/4$	20.	$(-100)\div 25$
21.	$(-75)\div(-25)$	22.	$-39\div 3$	23.	$144\div(-9)$	24.	$(-65)\div 13$
25.	$18\cdot(-5)$	26.	$\frac{-85}{5}$	27.	$(-65)\div(-13)$	28.	$\frac{-144}{-6}$
29.	$25\cdot(-8)$	30.	$(-35)(-4)$	31.	$-52/-4$	32.	$52/-4$
33.	$-36\div 4$	34.	$(-13)(-15)$	35.	$13(-15)$	36.	$(-84)\div(-6)$

4.4 Order of Operations

Powers (Exponents): $3^8 = \underbrace{3\cdot 3\cdot \ldots \cdot 3}_{8 \text{ times}} \Rightarrow$ repeated multiplication

Example 1 ***Multiplication is*** **repeated addition.**

$3\cdot 4 \Rightarrow 4+4+4 \Rightarrow 12$

$5\cdot(-2) \Rightarrow (-2)+(-2)+(-2)+(-2)+(-2) \Rightarrow -10$

$4\cdot 6 \Rightarrow$

$2\cdot(-7) \Rightarrow$

Example 2 ***Exponents indicate*** **repeated multiplication**

$3^4 \Rightarrow 3\cdot 3\cdot 3\cdot 3 \Rightarrow 81$

$(-5)^3 \Rightarrow (-5)\cdot(-5)\cdot(-5) \Rightarrow -125$

$(-7)^4 \Rightarrow$

$2^{10} \Rightarrow$

Example 3 **Simplify the following three expressions.**

$2+4\cdot 3^2+5\cdot 7$ $\qquad$ $4+3^2+4^2$ $\qquad$ $4\cdot 5^2-3\cdot 2^4$

Order of Operations

1. Grouping: Parentheses ( ) Brackets [ ] Braces { } numerator and denominator
2. Exponents (Powers)
3. Multiply or Divide in order of occurrence from left to right
4. Addition ~~or Subtraction~~

Example 4

$(6-8)^3+(7-4)^2$ $\qquad$ $\dfrac{3^2+4^2}{7-2}$

4.4 Order of Operations Homework

Simplify the following expressions.

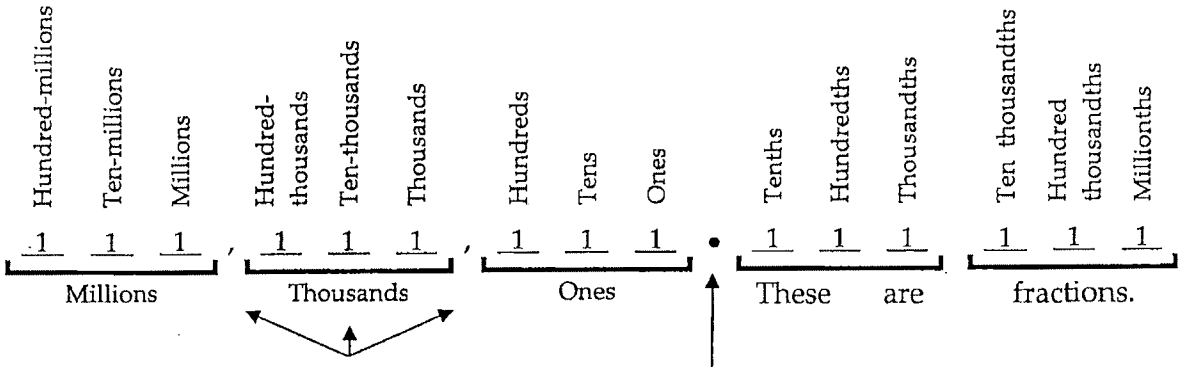
1. $(4-6)^2+(5-3)^3$
2. $5\cdot(2-8)+3\cdot(2+4)$
3. $5+3\cdot2^4$
4. $(3-1)^4+(1+2)^5$
5. $\dfrac{12^2+5^2}{15-2}$
6. $\dfrac{6^2-5^2}{22}$
7. $\left(\dfrac{2}{3}\right)^2+\left(\dfrac{4}{5}\right)^2$
8. $\dfrac{3\cdot(5^2-4^2)}{(5-2)^2}$
9. $4\cdot3^5-3\cdot2^7$
16. $\dfrac{3}{4}+\dfrac{2}{5}\cdot\dfrac{15}{7}$
17. $5\cdot(3-7)^2+2\cdot(5-2)^2$
18. $\dfrac{2^3}{5}+\dfrac{4}{7}$
20. $(3-5)^4\cdot(2+3)^2$
21. $\dfrac{2}{3}\cdot\left(\dfrac{4}{5}-\dfrac{3}{4}\right)$
23. $-3^4\cdot5+(-5)^2\cdot3$
26. $(-3)^4+(-5)^2$
27. $-3^4+(-5)^2$
28. $(-3)^4-5^2$
29. $(3\cdot8+5^2)-(3^2-4\cdot5)$
30. $\dfrac{8\cdot3-5\cdot6}{5^2-3^2}$
31. $-21-2+(-9)$
32. $-12-(-5)+(-3)$
33. $-2(-6)+7$
34. $-3+4(6-9)$
35. $-20\div(-5)(-4)$
36. $-9^2+(-5)^2$
37. $\dfrac{[35-63]}{[7-4\cdot2]}$
38. $-3-2(4-6)$
39. $\dfrac{[-6-(-3)]}{[1-2^2]}$
40. $-8+19-(-3)$
41. $-9-(-13)+(-12)-21$
42. $-\dfrac{5}{8}+\dfrac{3}{10}$
43. $-4(-9)+2$
44. $-3+4(-2)$
45. $-21.2(-14.5)$
46. $(-4)^2$
47. -5^2
48. $\left(\dfrac{2}{3}\right)^3$

If I have the belief that I can do it, I shall surely acquire the capacity to do it even if I may not have it at the beginning.

Mahatma Gandhi

Chapter 5 -- Decimals & Percents & Fractions

5.1 The Basics about Decimals



On the left side of the decimal, each group of three digits has a name and is separated by a comma.

A decimal is written between the whole numbers and fractions.

A decimal is either a proper fraction **or** **a decimal is a mixed number.**

.37 is the proper fraction $\frac{37}{100}$

4.29 is the mixed number $4\frac{29}{100}$

The mode is the number that occurs most often in a list.

Example 1: Find the mode for this list of numbers. 84, 90, 95, 98 and 88

Answer: The mode score for this list is _____.

Example 2: The sales of photo albums at Juanita's Card Shop for each day last week were $86, $149, $103, $118, $117, $126, $158 and $149. Find the mode.

Answer: The mode score is _____.

Steps for *Finding the Mode of a set of values (The value appearing the most times.)*

1. Find the value that appears most often in the list of values. **This is the mode.**
2. If no value appears more than once, **there is no mode.**
3. If two different values appear the same number of times, the list is **bimodal.**

Example 3: Find the mode of the following three sets of values:

3 11 3 2 10 9 18 3 Answer: _____

3 11 3 12 2 10 9 10 Answer: _____

3 1 11 12 2 10 9 15 7 Answer: _____

Summary:

The mean is the average and is found by first adding all values to get the total then dividing your answer by the number of values. $\textit{mean} = \frac{\text{add all numbers}}{\text{number of values}}$

The median is the middle number in a list of numbers when listed in order from smallest to largest.

The mode is the number that occurs most often in a list.

1.12 Mean, Median, Mode Homework

Find the mean of the following data.

1. Tanya had test scores of 96, 98, 88, 82, and 92. Find the mean (or average) score.
2. These are several months of utility bills: $25.12, $42.58, $76.19, $32, $81.11, $26.41, $19.76, $59.32, $71.18, and $21.03 Find the mean.
3. Find the mean for the following numbers: 74, 81, 39, 74, 82, 80, 100, 92, 74, 85.
4. What is the mean for these numbers: 51, 32, 49, 51, 49, 90, 49, 60, 17, 60?

Find the median of the following data.

5. Tanya had test scores of 96, 98, 88, 82, and 92.
6. Following is a list of monthly utility bills: $25.12, $42.58, $76.19, $32, $81.11, $26.41, $19.76, $59.32, $71.18, and $21.03
7. Find the median for the following numbers: 74, 81, 39, 74, 82, 80, 100, 92, 74, 85.
8. What is the median for these numbers: 51, 32, 49, 51, 49, 90, 49, 60, 17, 60?

Find the mean and the median of the following lists.

9. This is a list of math scores: 84, 90, 95, 98 and 88
10. Here is this list of measurements: 178 ft, 261 ft, 126 ft, 189 ft, 121 ft, and 195 ft.
11. This is a list of prices: $7, $23, $15, $6, $18, $12, $24
12. Here is a list of ages, in years: 74, 7, 15, 13, 25, 28, 47, 59, 32, and 68.

Find the mode for the following list of values.

13. Tanya had test scores of 96, 98, 88, 82, and 92.
14. Monthly utility bills of: $25.12, $42.58, $76.19, $32, $81.11, $26.41, $19.76, $59.32, $71.18, and $21.03
15. Find the mode for the following numbers: 74, 81, 39, 74, 82, 80, 100, 92, 74, 85.
16. What is the mode for these numbers: 51, 32, 49, 51, 49, 90, 49, 60, 17, 60?
17. This is a list of math scores: 84, 90, 95, 98 and 88
18. Here is this list of measurements: 178 ft, 261 ft, 126 ft, 189 ft, 121 ft, and 195 ft.
19. This is a list of prices: $7, $23, $15, $6, $18, $12, $24
20. Family member's ages, in years: 74, 7, 15, 13, 25, 28, 47, 59, 32, and 68.

Find the mean, the median, and the mode of the following lists.

21. A list of the age of part-time employees (in years): 28, 16, 22, 28, 34, 22, 28.
22. Total points on a health screening exam: 312, 219, 782, 312, 219, 426, 507, 600.
23. Monthly commissions of salespeople: $1706, $1289, $1653, $1892, $1301, $1782.
24. A set of numbers: 482, 485, 483, 485, 487, 487, 489, 486.
25. Monthly salary of a college president: $10,708; $11,519; $10,972; $12,546; $13,905; $12,182.
26. These are Maria Montaño's test scores in her arithmetic math course:

93 76 83 93 78 82 87 85

from arithmetic book fall 2009

Find the mode.

11. 26, 34, 43, 26, 51
12. 17, 7, 11, 11, 14, 17, 18
13. 0.2, 0.2, 1.7, 1.9, 2.4, 0.2
14. 700, 700, 800, 2700, 800
15. $14, $17, $21, $29, $17, $2
16. 20, 20, 20, 20, 20, 500
17. One summer, a student earned the following amounts over a four-week period: $102, $112, $130, and $98. What was the average amount earned per week? The median?
18. *Gas Mileage.* A 2001 Ford Focus gets 528 mi of highway driving on 16 gal of gasoline. What is the gas mileage?
19. To get an A in math, Marcus must score an average of 90 on four tests. His scores on the first on the first three tests were 94, 78, and 92. What is the lowest score that he can make on the

He who passively accepts evil is as much involved in it as he who helps to perpetrate it. He, who accepts evil without protesting against it, is really cooperating with it.

Martin Luther King, Jr.

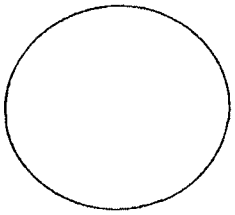
Chapter 2 – Fractions

2.1 Reducing (Simplifying) Fractions

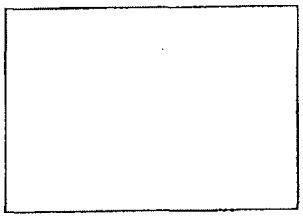
Drawing fractions

Fraction $\Rightarrow$ parts of a whole $\Rightarrow \frac{a}{b} \Rightarrow \frac{2}{3} \Rightarrow$ 2 parts out of 3 total

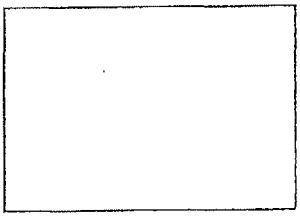
Example 1: **Draw** $\frac{2}{3}$ of the whole circle.

$\frac{2}{3} \Rightarrow$ 

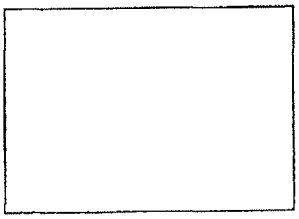
Example 2:
Show 4 parts out of a total of 6 below.
a total of 3.

$\frac{4}{6} \Rightarrow$ 

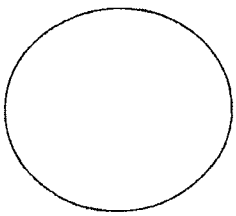
Example 3:
Show 8 parts out of a total of 12.

$\frac{8}{12} \Rightarrow$ 

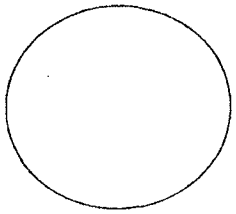
Example 4:
Show 2 parts out of

$\frac{2}{3} \Rightarrow$ 

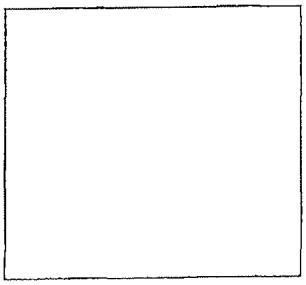
Reducing Fractions

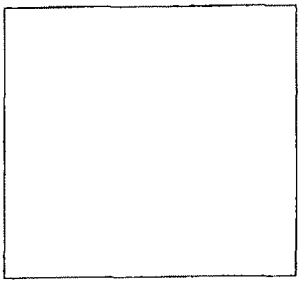


$\frac{2}{6}$



$\frac{1}{3}$

 $\Leftarrow \frac{}{8}$

 $\Leftarrow \frac{1}{2}$

Factors are numbers which are multiplied. For example: the factors of 6 are 2 times 3

We reduce fractions by reducing common factors between the numerator and denominator. This means that fractions like $\frac{2}{2}$ or $\frac{3}{3}$ are reduced to 1. What other fractions can be written as 1?

To reduce 2/6, 2 is written as the factors 2•1 and the 6 is written as the factors 2•3

$$\frac{2}{6} \Rightarrow \frac{2\cdot 1}{2\cdot 3} \Rightarrow \frac{1}{3}$$

To reduce 6/9, 6 is written as the factors 2•3 and the 9 is written as the factors 3•3

$$\frac{6}{9} \Rightarrow \frac{3\cdot 2}{3\cdot 3} \Rightarrow \frac{2}{3}$$

Example 5: **Reduce** these fractions by first writing in factored form then reduce to ones.

$\frac{4}{14} \Rightarrow$

$\frac{6}{15} \Rightarrow$

$\frac{35}{56} \Rightarrow$

$\frac{39}{52} \Rightarrow$

Reducing using primes

Prime numbers are numbers where the only factors are one and itself.

What are some of the prime numbers? _____ _____ _____ _____ _____ _____

Example 6: Use primes to reduce fractions.

Breaking numbers to prime form or using one prime at a time.

$\frac{36}{42}$ $\frac{36}{42}$

Example 7: Use primes to reduce fractions.

$\frac{24}{90}$

Example 8: Use primes to reduce fractions.

$\frac{45}{120}$

2.1 Reducing Fractions Homework Use primes to reduce fractions.

1. $\frac{8}{12}$ 2. $\frac{15}{25}$ 3. $\frac{45}{75}$ 4 $\frac{105}{280}$

5. $\frac{54}{90}$ 6. $\frac{210}{510}$ 7. $\frac{168}{216}$ 8. $\frac{270}{315}$

9. $\frac{182}{390}$ 10. $\frac{240}{400}$ 11. 12.

Rewrite the fractions so they have the indicated denominator.

13. $\frac{2}{5} \Rightarrow \frac{\quad}{15}$ 14. $\frac{2}{3} \Rightarrow \frac{\quad}{27}$ 15. $\frac{3}{7} \Rightarrow \frac{\quad}{42}$ 16. $\frac{4}{9} \Rightarrow \frac{\quad}{63}$

What is the fraction represented by the following?

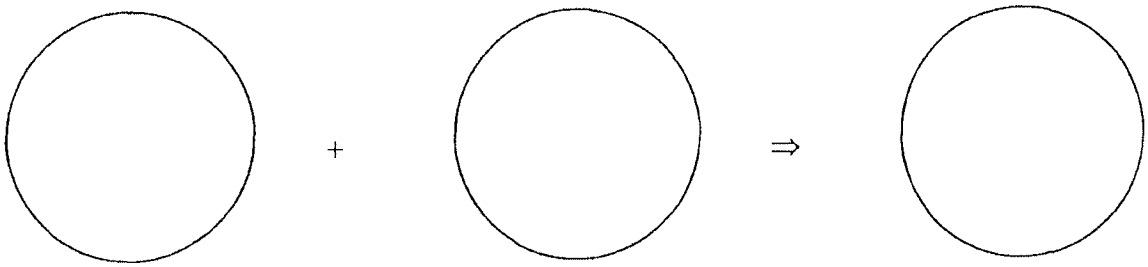
17. .5 18. .1 19. .25 20. .3

21. .75 22. .10 23. 50% 24. 75%

25. .50 26. 25%

2.2 Adding and Subtracting Fractions

To add or subtract fractions we must have a ***common denominator***.



$\frac{1}{4} + \frac{2}{4} \Rightarrow \frac{3}{4}$

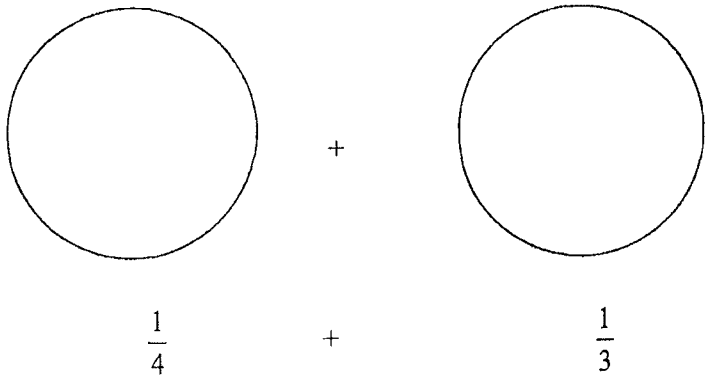
Formula for adding fractions with common denominators

$\frac{a}{c}+\frac{b}{c} \Rightarrow \frac{a+b}{c}$ Add the numerators and carry the common denominator.

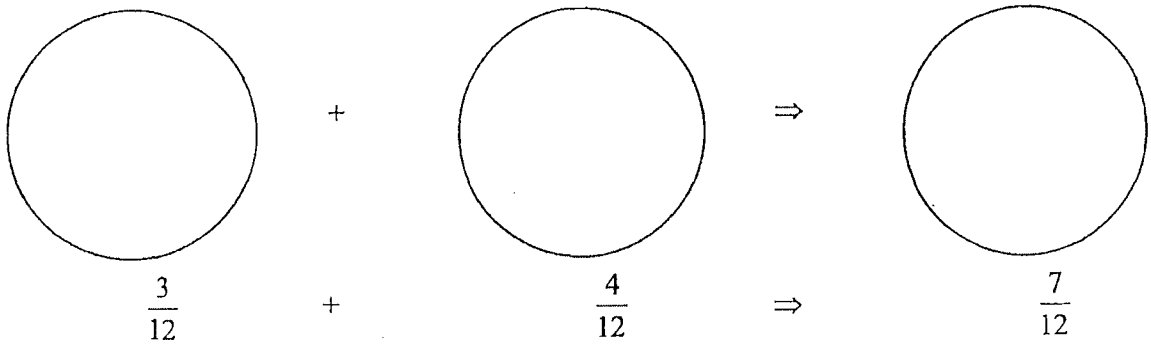
Example 1: $\frac{3}{8}+\frac{2}{8} \Rightarrow$

Example 2: $\frac{5}{12}+\frac{3}{12} \Rightarrow$

What if we do not have a common denominator?



$\frac{1}{4}$ + $\frac{1}{3}$



$\frac{3}{12}$ + $\frac{4}{12}$ $\Rightarrow$ $\frac{7}{12}$

Formula for adding fractions with different denominators
(Assuming there are no common factors in the denominator)

$$\frac{a}{b}+\frac{c}{d} \Rightarrow \frac{a}{b}\cdot\frac{d}{d} + \frac{c}{d}\cdot\frac{b}{b} \Rightarrow \frac{a\cdot d+b\cdot c}{b\cdot d}$$

multiply by the $\frac{\text{other denominator}}{\text{other denominator}}$

otherwise you need to multiply by the $\frac{\textit{lowest common denominator}}{\textit{lowest common denominator}}$

Example 3 Add the fractions
(no common factors in denominators)
$\frac{2}{5}+\frac{1}{4}$

Example 4 Add the fractions
(denominators with common factors)
$\frac{3}{8}+\frac{7}{12}$

Subtraction is exactly like addition. We will also *need a common denominator* to subtract.

Formula for subtracting fractions with common denominators

$$\frac{a}{c} - \frac{b}{c} \Rightarrow \frac{a-b}{c}$$

Formula for subtracting fractions with different denominators

$$\frac{a}{b} - \frac{c}{d} \Rightarrow \frac{a \cdot d - b \cdot c}{b \cdot d}$$

Simplify:

Example 5

$$\frac{11}{12} - \frac{3}{12}$$

Example 6

$$\frac{5}{6} - \frac{3}{8}$$

Example 7

$$\frac{7}{10} - \frac{4}{15}$$

2.2 Adding and Subtracting Fractions Homework

Preliminary work. Write the *prime* factors for the following numbers.

a. 8	b. 12	c. 6	d. 15	e. 14	f. 36	g. 2
h. 18	i. 52	j. 16	k. 32	l. 45	m. 78	n. 20
o. 19	p. 35	q, 40	r. 115	s. 51	t. 92	u. 56
v. 77	w. 28	x. 23	y. 49			

Add or subtract the fractions as indicated.

1. $\frac{3}{4}+\frac{1}{3}$	2. $\frac{5}{8}+\frac{1}{6}$	3. $\frac{5}{12}+\frac{7}{15}$	4. $\frac{2}{5}+\frac{3}{7}$
5. $\frac{3}{10}+\frac{2}{9}$	6. $\frac{4}{5}-\frac{1}{6}$	7. $\frac{3}{8}-\frac{2}{10}$	8. $\frac{9}{13}-\frac{3}{14}$
9. $\frac{5}{7}-\frac{2}{9}$	10. $\frac{7}{18}-\frac{4}{21}$	11. $\frac{13}{21}+\frac{5}{18}$	12. $\frac{8}{15}-\frac{4}{25}$
13. $\frac{12}{17}-\frac{5}{19}$	14. $\frac{8}{15}-\frac{3}{20}$	15. $\frac{5}{24}+\frac{7}{20}$	16. $\frac{23}{52}+\frac{17}{78}$
17. $\frac{5}{16}-\frac{3}{12}$	18. $\frac{9}{35}+\frac{8}{21}$	19. $\frac{5}{12}+\frac{7}{16}$	20. $\frac{14}{25}-\frac{6}{35}$
21. $\frac{5}{16}+\frac{7}{20}$			

2.3 Multiplying and Dividing Fractions

Fraction $\Rightarrow \frac{a}{b} \Rightarrow a \div b$ Fractions can be read like the division of two numbers.

Multiplying Fractions (Assuming there are no common factors in the denominator)
Multiply the numerators and multiply the denominators.

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}$$

Otherwise,

1. **Factor** the numerator and the denominator
2. **Reduce** common factors to one
3. **Multiply** the numerators and multiply the denominators.

Simplify:

Example 1:

$\frac{3}{5}\cdot\frac{2}{6}$

Example 2:

$\frac{5}{6}\cdot\frac{3}{8}$

Example 3:

$\frac{12}{35}\cdot\frac{25}{42}$ Or $\frac{12}{35}\cdot\frac{25}{42}$

Example 4:

$\frac{4}{21}\cdot\frac{9}{20}$

Example 5:

$\frac{12}{39}\cdot\frac{26}{63}$

Dividing Fractions ⇒ Multiplying by the reciprocal

$\frac{a}{b} \div \frac{c}{d}$ Or $\frac{a/b}{c/d} = \frac{a}{b} \cdot \frac{d}{c}$

↗ ↑ ↑

copy change flip

$\frac{a}{b} \cdot \frac{d}{c}$ **Now do the multiplication.**

Example 6: Simplify

$\frac{5}{6} \div \frac{3}{8}$

Example 7: Simplify

$\frac{14}{15} \div \frac{24}{35}$

First: **copy-change-invert**

Second: **factor** the numerator and the denominator then **reduce** to one where possible.

Third, **multiply**.

Example 8:

$\frac{12}{35} \div \frac{25}{42}$

2.3 Multiplying and Dividing Fractions Homework

Perform the indicated operations on the fractions

1. $\frac{3}{4} \cdot \frac{5}{6}$
2. $\frac{4}{9} \cdot \frac{6}{20}$
3. $\frac{12}{35} \cdot \frac{21}{24}$
4. $\frac{13}{15} \cdot \frac{10}{39}$
5. $\frac{14}{23} \cdot \frac{5}{21}$
6. $\frac{3}{22} \cdot \frac{11}{15}$
7. $\frac{6}{55} \cdot \frac{10}{33}$
8. $\frac{3}{5} \cdot \frac{4}{7}$
9. $\frac{20}{21} \cdot \frac{35}{32}$
10. $\frac{34}{49} \cdot \frac{21}{51}$
11. $\frac{22}{45} \cdot \frac{25}{33}$
12. $\frac{15}{23} \cdot \frac{12}{35}$
13. $\frac{2}{3} \div \frac{4}{7}$
14. $\frac{5}{8} \div \frac{3}{14}$
15. $\frac{8}{15} \div \frac{3}{25}$
16. $\frac{24}{35} \div \frac{8}{21}$
17. $\frac{26}{45} \div \frac{39}{40}$
18. $\frac{12}{25} \div \frac{9}{35}$
19. $\frac{46}{50} \div \frac{115}{24}$
20. $\frac{34}{48} \div \frac{51}{92}$
21. $\frac{42}{55} \div \frac{34}{77}$
22. $\frac{15}{28} \div \frac{12}{35}$
23. $\frac{14}{25} \div \frac{35}{56}$
24. $\frac{5}{18} \div \frac{15}{28}$

2.4 Mixed Numbers

mixed number $\Leftrightarrow$ improper fraction

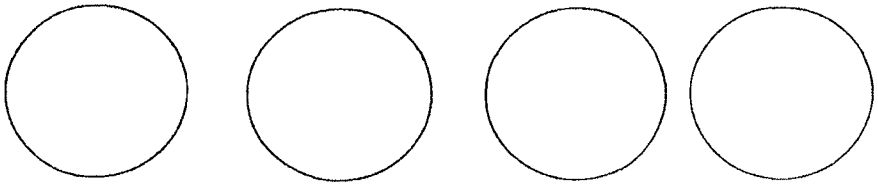
Fraction $\Rightarrow \dfrac{\text{numerator}}{\text{denominator}}$

Proper fraction $\Rightarrow$ numerator is smaller than the denominator $\frac{2}{3}$

Improper fraction $\Rightarrow$ numerator larger or same size as denominator $\frac{3}{3}$ or $\frac{5}{3}$

Mixed number $\Rightarrow$ whole number together with a fraction $5\frac{2}{7}$

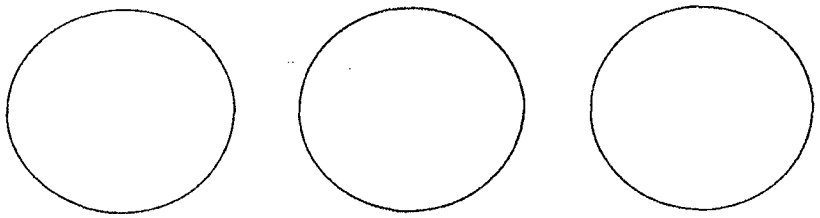
Drawing **Example 1** Split each whole circle into 4 equal pieces and shade 13 pieces.



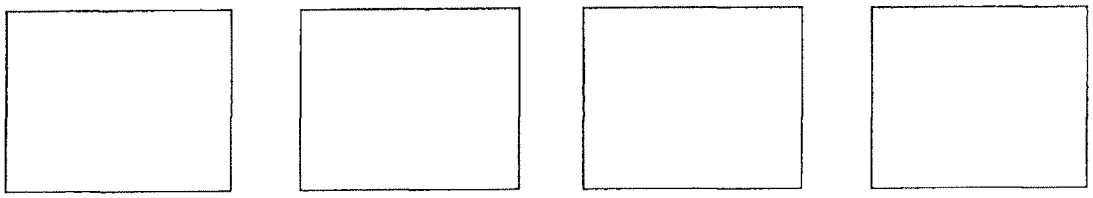
Each piece is $\frac{1}{4}$. There are 13 pieces shaded $\Rightarrow \frac{13}{4}$.

There are 3 wholes shaded and one-fourth of a whole $\Rightarrow 3\frac{1}{4}$.

Example 2 Draw five halves and write this two ways: as a fraction and a mixed number.



Example 3 Draw $3\frac{2}{5}$ and write as a fraction.



Converting from a *mixed number* to an improper fraction. $4\frac{3}{8}$

1. Multiply the whole number by the denominator. $\rightarrow$ 4(8)
2. Add the numerator; this is the new numerator) $\rightarrow$ 32+3
3. Write as an improper fraction. $\dfrac{\textit{new numerator}}{\textit{denominator}}$ $\rightarrow \dfrac{35}{8}$

Example 4 Write each mixed number as an improper fraction.

$5\frac{2}{3}$ $3\frac{4}{5}$ $2\frac{3}{7}$

Steps for converting (changing) from an *improper fraction* to a mixed number.

1. Divide the numerator by the denominator. (Long Division) $\frac{33}{6}$

$$denominator\overline{)numerator}^{quotient} \qquad 6\overline{)33}$$

2. Write as a fraction.

$$quotient\ \frac{remainder}{denominator} \qquad 5\frac{3}{6} \text{ or } 5\frac{1}{2}$$

Example 5 Write each improper fraction as a mixed number.

$\frac{17}{5}$ $\frac{34}{7}$ $\frac{63}{5}$ $\frac{234}{11}$

2.4 Mixed Numbers Homework

Convert to mixed number form.

1. $\frac{8}{3}$ 2. $\frac{9}{4}$ 3. $\frac{16}{5}$ 4. $\frac{35}{8}$ 5. $\frac{25}{7}$

6. $\frac{75}{12}$ 7. $\frac{14}{5}$ 8. $\frac{125}{7}$ 9. $\frac{39}{9}$ 10. $\frac{335}{17}$

Convert to improper fractions

11. $3\frac{4}{5}$ 12. $5\frac{3}{7}$ 13. $2\frac{5}{9}$ 14. $9\frac{3}{4}$ 15. $5\frac{2}{3}$

16. $12\frac{5}{6}$ 17. $3\frac{7}{10}$ 18. $12\frac{5}{11}$ 19. $9\frac{3}{8}$ 20. $23\frac{5}{9}$

Reduce and convert the fractions

21. $\frac{25}{15}$ 22. $\frac{270}{126}$ 23. $\frac{52}{12}$ 24. $\frac{90}{75}$ 25. $\frac{24}{16}$

26. $\frac{225}{100}$ 27. $\frac{56}{21}$ 28. $\frac{84}{72}$ 29. $\frac{150}{36}$ 30. $\frac{234}{52}$

2.5 Operations with Mixed Numbers

The four main operations are _____ _____ _____ _____

Steps for Adding Mixed Numbers or subtracting mixed numbers

1. Convert mixed numbers to improper fractions. or
2. Perform the operation.
3. Convert back to a mixed number, if necessary.

1. Add the whole numbers
2. Add the fractions
3. Write the answer as a mixed number.

Example 6: Simplify

$2\frac{3}{5}+4\frac{1}{6} \Rightarrow \frac{13}{5}+\frac{25}{6}$

$\Rightarrow \frac{13\cdot 6+5\cdot 25}{5\cdot 6} \Rightarrow \frac{78+125}{30} \Rightarrow \frac{203}{30}$

$30\overline{)203}$ $\quad 6$

$-180 \quad \Rightarrow 6\frac{23}{30}$

23

$2\frac{3}{5}+4\frac{1}{6}$

$2+4+\frac{3}{5}+\frac{1}{6}$

$6+\frac{3\cdot 6}{5\cdot 6}+\frac{1\cdot 5}{6\cdot 5}$ $\qquad$ 18+5=23

$6+\frac{23}{30} \quad \Rightarrow 6\frac{23}{30}$

Example 7: Simplify

$6\frac{2}{15}-2\frac{1}{6}$

$6+\frac{2}{15}-2-\frac{1}{6}$

$4+\frac{2}{15}-\frac{1}{6}$ $\qquad$ $\frac{3\cdot 5}{}$ $\quad$ $\frac{2\cdot 3}{}$ $\qquad$ LCD is $3\cdot 5\cdot 2 = 30$

$4+\frac{2}{15}\cdot\frac{6}{6}-\frac{1}{6}\cdot\frac{5}{5}$

$4+\frac{12}{30}-\frac{5}{30}$

$4+\frac{7}{30}$

$4\frac{7}{30}$

Example 8: Simplify

$9\frac{5}{12}-3\frac{4}{15}$

Example 9 **Simplify the expressions**

$1 - \frac{2}{5}$ $3 - \frac{3}{4}$ $8 - 2\frac{1}{4}$ $8 + 2\frac{1}{4}$

Multiplying or Dividing Mixed Numbers Note: First rewrite the mixed numbers as fractions.

Example 10

$2\frac{3}{5} \cdot 4\frac{1}{6}$

Example 11

$2\frac{3}{5} \div 4\frac{1}{6}$

Example 12 Combine the mixed numbers by performing the indicated operations.

$6\frac{5}{8} \div 3\frac{7}{12}$ $7\frac{5}{6} - 4\frac{3}{8}$

2.5 Operations with Mixed Numbers Homework

Perform the operations indicated .

1. $3\frac{1}{4} + 2\frac{1}{3}$	2. $5\frac{2}{3} + 1\frac{3}{5}$	3. $6\frac{2}{5} - 3\frac{1}{4}$	4. $2\frac{2}{7} - \frac{3}{4}$
5. $3\frac{5}{6} + 2\frac{3}{8}$	6. $2\frac{1}{4} \cdot 3\frac{2}{5}$	7. $5\frac{1}{4} \cdot 3\frac{1}{7}$	8. $7\frac{1}{6} \cdot 4\frac{2}{5}$
9. $3\frac{1}{7} \div 2\frac{4}{5}$	10. $12\frac{2}{3} \div 8\frac{4}{9}$	11. $2\frac{5}{6} \div 3\frac{5}{8}$	12. $8\frac{4}{5} \div 2\frac{4}{15}$
13. $1\frac{13}{15} \div 1\frac{17}{25}$	14. $7\frac{2}{7} \div 3\frac{5}{21}$	15. $6\frac{1}{4} + 5\frac{5}{6}$	16. $3\frac{7}{12} - 1\frac{9}{16}$
17. $2\frac{5}{14} - 1\frac{21}{34}$	18. $7\frac{1}{22} \cdot 5\frac{2}{15}$	19. $3\frac{4}{15} \cdot 3\frac{17}{21}$	20. $6\frac{9}{14} \div 8\frac{4}{7}$

2.6 Form A: Addition and Subtraction

1. $3\frac{1}{2}+8\frac{3}{4}+5\frac{3}{4}$

2. $\frac{8}{3}-\frac{1}{9}+\frac{5}{7}$

3. $2\frac{1}{3}-8\frac{3}{4}+5$

4. $8+13\frac{3}{27}-\frac{1}{9}$

5. $\frac{7}{3}-8\frac{3}{4}+5$

6. $8+13\frac{7}{2}-\frac{1}{9}$

7. $\frac{5}{9}+\frac{2}{5}-2\frac{4}{5}$

8. $6\frac{2}{5}-3\frac{2}{5}+9\frac{2}{5}$

9. $1\frac{1}{6}+\frac{3}{8}$

10 $24-3\frac{4}{11}$

11. $1\frac{1}{6}+\frac{3}{4}$

12. $\frac{5}{9}+\frac{8}{3}-\frac{14}{5}$

13. $\frac{32}{5}-3\frac{2}{5}+\frac{47}{5}$

14. $8\frac{1}{4}+3\frac{5}{6}$

15. $25-25\frac{2}{3}+\frac{8}{7}$

16. $\frac{2}{3}+\frac{1}{19}-\frac{5}{2}$

17. $\frac{3}{5}-9+\frac{18}{7}$

18. $3\frac{1}{2}+8\frac{3}{4}+5\frac{3}{4}$

19. $2\frac{2}{3}-\frac{1}{9}+\frac{5}{6}$

20. $8\frac{1}{4}+3\frac{5}{6}$

21. $\frac{2}{3}+\frac{1}{9}-2\frac{1}{2}$

22. $\frac{3}{5}-9+\frac{5}{7}$

2.7 Form B: Multiplication and Division

1. $5\frac{4}{9}\cdot 8\frac{4}{9}$
2. $\frac{17}{3}\div\frac{88}{121}$
3. $5\frac{3}{7}\div\frac{2}{63}$
4. $\frac{15}{42}\div\frac{225}{18}$
5. $4\frac{7}{11}\div 2\frac{1}{2}$
6. $5\frac{4}{9}\cdot\frac{33}{2}$
7. $11\frac{1}{2}\cdot 5\frac{1}{4}$
8. $\frac{80}{140}\div\frac{22}{133}$
9. $6\frac{1}{2}\div\frac{3}{4}$
10. $2\frac{3}{5}\cdot 1\frac{4}{7}$
11. $\frac{4}{7}\div 1\frac{5}{8}$
12. $-\frac{5}{7}\cdot\frac{49}{65}$
13. $13\frac{3}{5}\cdot 8\frac{3}{2}$
14. $\frac{80}{140}\div\frac{135}{22}$
15. $5\frac{1}{2}\cdot 2\frac{2}{5}$
16. $-2\frac{1}{3}\div\frac{-2}{3}$
17. $\frac{4}{35}\cdot\frac{5}{24}$
18. $\frac{-3}{8}\div\frac{-6}{32}$
19. $5\frac{4}{9}\cdot 8\frac{4}{9}$
20. $\frac{7}{3}\div\frac{88}{121}$
21. $\frac{31}{2}\cdot\frac{12}{5}$
22. $\frac{7}{-3}\div\frac{-2}{3}$
23. $\frac{4}{35}\cdot\frac{5}{24}$
24. $\frac{-3}{8}\div\frac{-6}{32}$

2.8 Form C: Mixed Problems

1. $\frac{7}{3}+12\frac{4}{9}\cdot\frac{5}{4}$
2. $\frac{17}{2}\div\frac{8}{3}+18$
3. $\frac{1}{5}+7\frac{1}{3}\cdot 9$
4. $\frac{8}{3}+3\frac{5}{6}\cdot\frac{15}{4}$
5. $\frac{7}{13}-4\frac{7}{6}\div 2\frac{1}{2}+5\frac{1}{4}$
6. $6\frac{1}{2}\div 2\frac{2}{3}-4\frac{1}{3}$
7. $5\frac{2}{5}-6\frac{1}{9}\cdot\frac{1}{5}$
8. $\frac{19-4}{3+7}\div 1\frac{5}{8}$
9. $\frac{1-9}{7+5}\cdot\frac{57}{3}$
10. $8\frac{3}{2}\div 2\frac{2}{3}-7\frac{4}{7}$
11. $\frac{27}{5}-6\frac{9}{8}\cdot\frac{39}{2}$
12. $\frac{2}{3}+1\frac{1}{2}\cdot 6\frac{1}{3}$
13. $2\frac{1}{3}\div\frac{1}{4}+3$
14. $\frac{8}{15}+\frac{3}{2}\cdot\frac{19}{3}$
15. $\frac{1}{6}+1\frac{9}{2}\div 14$
16. $\frac{2}{5}+2\frac{1}{2}\cdot\frac{1}{7}$
17. $\frac{2}{51}\div\frac{8}{3}+18$
18. $\frac{8}{15}+\frac{3}{2}\cdot\frac{19}{3}$
19. $\frac{16}{3}\div\frac{4}{33}+\frac{18}{3}$
20. $\frac{1}{3}+\frac{3}{2}\cdot 6\frac{1}{3}$
21. $\frac{1}{6}+5\frac{1}{2}\div 7$

2.9 Ratios

A ratio compares one number with another. Write ratios as fractions reduced to lowest terms.

Ratios can be written in several different ways. If we compare the number 6 with the number 7, the ratio is read as "6 to 7" and is written $\frac{6}{7}$ or 6:7 or 6 to 7. We are going to use the fraction style.

If two numbers with units are being compared (like 7 inches to 1 foot), **it is important to state the units of the numerator and the denominator clearly:** $\frac{7\ inches}{1\ foot}$

In fact, if possible, make the units the same: $\frac{7\ \text{inches}}{12\ \text{inches}}$ You are comparing the same unit.

Remember that ratios remain fractions but must be reduced to lowest terms.

Example 1: Compare the number 20 with the number 37.

Answer: The ratio is

Example 2: Compare 1 dollar to 5 quarters.

Answer: The ratio is

Example 3: What is the ratio of 12 inches to 15 inches?

Answer: The ratio is

Example 4: What is the ratio of $\frac{4}{5}$ to $\frac{8}{3}$?

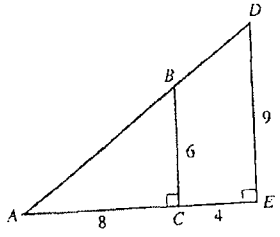
2.9 Ratios Homework

Give the ratio of the following quantities.

1. 8 to 6
2. 6 to 8
3. 64 to 12
4. 12 to 64
5. 100 to 250
6. 250 to 100
7. 13 to 26
8. 36 to 18
9. $\frac{3}{4}$ to $\frac{1}{4}$
10. $\frac{5}{8}$ to $\frac{3}{8}$
11. $\frac{7}{3}$ to $\frac{6}{3}$
12. $\frac{9}{5}$ to $\frac{11}{5}$
13. $\frac{6}{5}$ to $\frac{6}{7}$
14. $\frac{5}{3}$ to $\frac{1}{3}$
15. $2\frac{1}{2}$ to $3\frac{1}{2}$
16. $5\frac{1}{4}$ to $1\frac{3}{4}$
17. $2\frac{2}{3}$ to $\frac{5}{3}$
18. $\frac{1}{2}$ to $3\frac{1}{2}$
19. 0.05 to 0.15
20. 0.21 to 0.03
21. 0.3 to 3
22. 0. 5 to 10
23. 1.2 to 10
24. 6.4 to 0.8
25. $\frac{1}{2}$ to 1.5
26. $\frac{1}{4}$ to 0.75

27. Regarding the diagram below, AC represents the length of the line segment that starts at A and ends at C.

From the diagram we see that AC = 8.

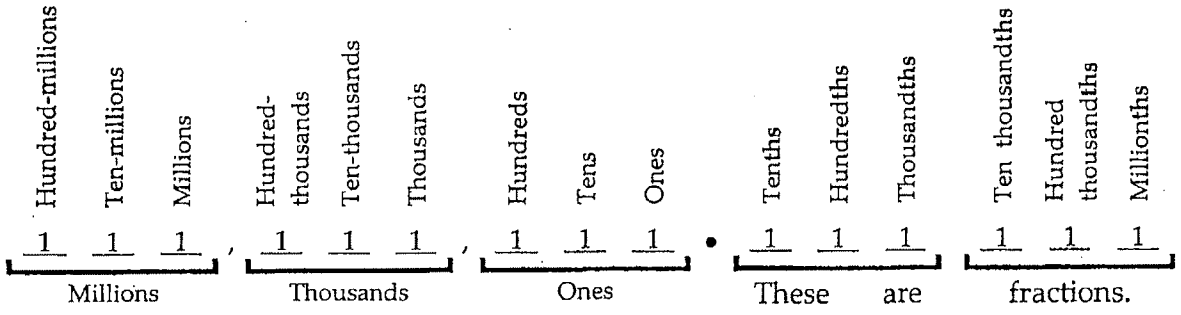


a. Find the ratio of BC to AC
b. What is the length AE?
c. Find the ratio of DE to AE.

28. A family of four budgeted $400 for food, $100 for gas, $150 for utilities, and $250 for the house payment.

a. What's the ratio of the house payment to the food bill?
b. What is the ratio of the gas bill to the food bill?
c. What's the ratio of the utilities bills to the food bill?
d. What is the ratio of the house payment to the utilities bills?

A decimal written in fraction form has a denominator with a power of ten.



• _____ _____ _____ _____ _____

$\frac{1}{10}$ $\frac{1}{10^2}$ or $\frac{1}{100}$ $\frac{1}{10^3}$ or $\frac{1}{1{,}000}$ $\frac{1}{10^4}$ or $\frac{1}{10{,}000}$ $\frac{1}{10^5}$ or $\frac{1}{100{,}000}$

ten*ths* hundred*ths* thousand*ths* ten-thousand*ths* hundred-thousand*ths*

The exponent tells how many zeros are in the number. In 10^2 the exponent is 2 and the 100 has 2 zeros.

tenths *hundred**ths*** *thousand**ths*** *ten-thousand**ths*** *hundred-thousand**ths***

The location of the number (based on the decimal point) gives the value of the number.

The denominator is a power of 10 i.e. 10, 10^2, 10^3, 10^4, 10^5, etc.

For example: *the 5 goes under the 10 in the number* five-tenths • **5**

The 7 goes under the 100 in the number seven-hundredths • **0 7**

Example: Write the number forty-three thousandths •

Write the number twenty-five hundred**ths.** •

Write the number two hundred forty-three **and** eighteen ten-thousand**ths.** •

Write the number one thousand thirty-six **and** ninety-five thousand**ths.** •

Write the number 37 and 50 hundred**ths.** •

Read the number **1 4 5 7 • 0 6 3**

Answer: one thousand four hundred fifty-seven and sixty-three thousandths.

5.1 The Basics about Decimals Homework

Write each fraction as a decimal or a mixed number.

1. $\frac{7}{10}$ 2. $\frac{9}{10}$ 3. $\frac{27}{100}$ 4. $\frac{39}{100}$ 5. $\frac{2}{5}$ 6. $\frac{4}{5}$ 7. $5\frac{2}{5}$ 8. $6\frac{4}{5}$ 9. $-\frac{29}{500}$

10. $-\frac{3}{125}$ 11. $7\frac{1}{125}$ 12. $9\frac{1}{200}$ 13. $15\frac{401}{500}$ 14. $11\frac{203}{500}$ 15. $\frac{601}{2000}$ 16. $\frac{1003}{5000}$

17) $\frac{6}{10}+\frac{4}{100}+\frac{3}{1,000}+\frac{8}{10,000}$ 18) $\frac{3}{10}+\frac{0}{100}+\frac{8}{1,000}+\frac{4}{10,000}$ 19) $\frac{9}{100}+\frac{3}{1,000}$

20) $68+\frac{3}{10}+\frac{8}{100}+\frac{4}{1,000}+\frac{6}{10,000}$ 21) $491+\frac{0}{10}+\frac{3}{100}+\frac{6}{1,000}+\frac{8}{10,000}$

22) $436+\frac{0}{10}+\frac{0}{100}+\frac{0}{1,000}+\frac{0}{10,000}$ 23) $\frac{483}{10,000}$ 24) $\frac{8}{1,000}+\frac{2}{10}$

Write each number in words.

25. 2.8 26. 5.1 27. 9.08 28. 12.06 29. −705.625 30. −804.399 31. 0.0046

32. 0.0083 33. 5.62 34. 9.57 35. 16.23 36. 47.65 37. −.205 38. −.495

39. 167.009 40. 233.056 41. 3000.04 42. 5000.02 43. 105.6 44. 410.3

Write the words as numbers

45. four hundred ten and three tenths

46. Two and eight tenths

47. Five and one tenth

48. Nine and eight hundredths

49. Twelve and six hundredths

50. Negative seven hundred five and six hundred twenty-five thousandths

51. Negative eight hundred four and three hundred ninety-nine thousandths.

52. Forty-six ten-thousandths.

53. Eighty-three ten thousandths

54. Five and sixty-two hundredths

55. nine and fifty seven hundredths

56. sixteen and twenty-three hundredths

57. forty-seven and sixty five hundredths

58. Negative two hundred five thousandths

59. negative four hundred ninety-five thousandths

60. one hundred sixty-seven and nine thousandths

61. two hundred thirty-three and fifty-six thousandths

62. three thousand and four hundredths

63. five thousand and two hundredths

64. one hundred five and six tenths

65. Read the following numbers and then write them in exponent form where 10 is the base:
10 100 1,000 10,000 100,000 1,000,000

66. Multiply the following and see if you can see any pattern.
3.5(100) .04(10) 27.396(1000) 142.789(100) 5.0432(10)

Read the following numbers and write them as fractions or mixed numbers.

67) .6438 68) .3084 69) .093 70) 68.3846

71) 491.0368 72) 436.0000 73) .0483 74) .208

5.2 Changing Decimals, Fractions, and Percents

Changing from *decimal* to fraction is easy since all you do is write the number as it is read, then reduce the fraction where possible.

Example .5 is read as five-tenths.

So it is written as the fraction $\frac{5}{10}$ and then reduced to $\frac{1}{2}$

Changing from *decimal* to percent is also easy since **percent** means **per 100.**

This means that in percent the denominator is always 100.

50% is $\frac{50}{100}$ or $\frac{1}{2}$ 100% is $\frac{100}{100}$ or 1 100% is the same as the number 1.

Example 1 .25 is read as twenty-five hundredths. So it is written as the fraction $\frac{25}{100}$.
This is 25 per 100; that is 25 percent which is 25%.
.25 (100%) = 25%

Example 2 Change .72 to a percent.
Answer: .72 is read as _____. So it is written as the fraction _____.

This is _____ which is _____%

Example 3 Change .385 to a percent.
.385 times 100% (.385 times 100)%

Changing from *percent* to fraction

Changing from *percent* to decimal

Changing from *fraction* to decimal

Changing from *fraction* to percent

Decimal	Fraction	Percentage
1	= 1.	= 100%
½	= .5	= 50%
$\frac{1}{3}$	= .33$\frac{1}{3}$	= 33$\frac{1}{3}$%
$\frac{1}{4}$	= .25	= 25%
$\frac{1}{5}$	= .2	= 20%
$\frac{1}{8}$	= .125	=12.5%

5.2 Changing Decimals, Fractions, and Percents *Homework*

Write the following **decimals** as fractions.

1. .28 2. .3171 3. 97.08 4. .081

5. 4.365 6. 36.005 7. 4.3 8. 14.3684

Write each **percent** as a decimal

9. 65%	10. 125%	11. 1.2%	12. 60%	13. 0.6%	14. 0.8%		
15. 22%	16. 44%	17. 530%	18. 270%	19. 5.6%	20. 1.9%		
21. 28%	22. 11.15%	23. 300%	24. 900%	25. 70%	26. 80%		

Write each **decimal** as a percent\

27. 0.41	28. 0.62	29. 0.06	30. 0.03	31. 1.00 or 1	32. 1.36		
33. 0.736	34. 0.457	35. 0.028	36. 0.014	37. 0.006	38. 0.009		
39. 3.00 or 3	40. 5.00 or 5	41. 0.3258	42. 0.7218				

Write each **fraction** as a percent

43. $\frac{2}{25}$ 44. $\frac{11}{50}$ 45. $\frac{1}{25}$ 46. $\frac{1}{50}$ 47. $\frac{9}{200}$ 48. $\frac{3}{40}$ 49. $\frac{7}{4}$ or $1\frac{3}{4}$

50. $\frac{11}{4}$ or $2\frac{3}{4}$ 51. $\frac{1}{16}$ 52. $\frac{7}{80}$ 53. $\frac{31}{300}$ 54. $\frac{31}{400}$ 55. $\frac{179}{800}$ 56. $\frac{7}{32}$

5.3 Adding & Subtracting Decimals

When **Adding (or Subtracting) Decimals**, line up the decimal point and the numbers according to their place value.

5.3 Adding & Subtracting Decimals Homework

Add the following decimal numbers.

1. 2.1 + 4.0 + 5.6 + 7.7
2. 64.38 + 7.76 + 3.1 + 629.34
3. 154.58 + 22.24 + 145.06 + 1.98 + 28.83
4. 234.12 + 86.3 + 2.71 + 81.67 + 950.33 + 822 + 9.76
5. 1.0719 + 17.0034 + 69.901 + 6.3027 + 45.4545 + 4.9669
6. 35.6825 + 4.3678 + 15.6293 + 23.8471 + 3.6325 + 49.1873
7. 651.07 + .4592 + .893 + 259.24 + 6.2432
8. 7.534 + 77.234 + 362.28 + 1.2457 + 32.567
9. 138.62 + .46235 + 1.4928 + 16.368 + 29832
10. 3.721 + 4.93 + 187.45 + 9.367 + .673 + 1.0079 + 14.7
11. 779.642 + 9.67 + 43.984 + .3468 + 12.56 + 687.29 + 30.09
12. 698.37 + 2.9482 + .32164 + 2984.5 + 84.375
13. 543.7 + 98.07645 + 2345 + 34 + .2 + 16.9 + 87.87
14. 2413.35 + 8096 + 2 + .0000865 + 12.006 + 90.09
15. .0000000007 + 3 + 423.65 + 900.0006
16. 132435.46 + 6342.35124 + 7452 + 978.142 + 13
17. 24.3 + 35.6 + .008612 + 1 + 1.89 + 6.80 + 73
18. 14423.3 + 5598.7098 + 25 + .0008 + 3.0001 + .01
19. 235.5555 + 34.444 + 3.23 + 4.1 + 5 + 69875
20. 55.698 + 2 + 24536.1 + 985674 + 32.6 + 12
21. 3542 + 6856 + 37764 + .000008 + .23
22. 33.32 + .9900001 + 768.49 + 3
23. 78.99999 + .2 + 365243 + 32 + .00002 + 78.1
24. 54.98 + .01 + 57 + 387967 + 100000032 + .3

Subtracting Decimals

Subtract the following decimal numbers.

1. 13.5836 – 8.7397
2. 252.3 – 18
3. 600 – 507.36
4. 435.6 – 21.54
5. 35.007 – 13.06
6. 200.4 – 87.895
7. 5194.362 – 486.089
8. 823.106 – 507.836
9. 89.9 – 24.3
10. 94.5 – 47.8
11. 45 – 14.78
12. 39.65 – 23.95
13. 283.78 – 36.89
14. 36 – 24.91
15. 436.275 – 9.839
16. 647.1 – 347.869
17. 16.4 – 12.625
18. 12.9 – 3.1
19. 4 – .0001
20. .0098 – .00006
21. 534.0009 – 2.09
22. 65.89 – 43.0008
23. 4.1 – .00978
24. 6 – 1.423

5.4 Multiplying Decimals

Multiplying Decimals Multiplying decimal numbers and multiplying whole numbers is the same except for the answer's decimal point placement. Count the number of digits to the right of the decimal in the first factor and in the second factor. The sum of these digits is the number of digits to the right of the decimal point in the answer.

Place Value

Fraction denominators and decimal denominators

Example Multiply (3.432)(1.2) There are three digits in the first factor and one digit in the second factor. This means there must be four (4) digits to the right of the decimal in the answer.
(3.432)(1.2) is 4.1184

Example Multiply (.1206)(.0034) answer is .00041004

5.4 Multiplying Decimals Homework

Multiply the following decimal numbers.

1. 23 x 2.51	2. 2.3 x .251	3. .003 x 26.32
4. 0.234 x 5.5	5. 36.2 x 0.76	6. 3.926 x .128
7. 3 x .835	8. .2 x 234	9. 5.24 x 1.97
10. 42.8 x 200	11. 24.6 x 1.004 x 2.007	12. 32 x 0.12 x 43.1
13. 65.009 x 2.41	14. 834.1 x 23 x 87.9	15. 1234 x .1234
16. 543.21 x 54	17. 3.4 x .009 x 12.5	18. 76.2 x 43 x 2.1
19. 54.3 x 234 x 2.3	20. 87 x 88 x .1	21. 534.5 x 22 x .312
22. 64.7 x .0008 x .08	23. 55.3 x 2345	24. 624.46 x .21 x .008

5.5 Dividing Decimals

Dividing Decimals The divisor must be a whole number. If it is not, then move the decimal in the divisor to the right and move the decimal in the dividend the same number of places to the right. Next place the decimal point for the answer directly above the newly located decimal point in the dividend. Now divide using the steps for long division.

Example: Divide $.21\overline{)4.321}$ Answer: 20.57

5.5 Dividing Decimals Homework

Divide the following decimals and carry your answer at least two decimal places.

1. $6.82 \div .21$	2. $.8402 \div 6.2$	3. $824 \div .21$	4. $326.2 \div 20.4$
5. $93.66 \div .602$	6. $828.3 \div 6.42$	7. $7.832 \div 42.6$	8. $1.72 \div 21$
9. $9.865 \div 3.48$	10. $106.82 \div 3.971$	11. $98.6 \div .83$	12. $483 \div 2.6$
13. $876.987 \div 4.32$	14. $.0072 \div 2.09$	15. $.0009 \div 33$	16. $5425.6895 \div 643.54$
17. $46.24 \div 35.2$	18. $867.0083 \div 65.781$	19. $764.98 \div 23.65$	20. $425.9978 \div .0054$

5.6 Percent Problems

1. What is 32% of 84?

2. What percent of 90 is 18?

3. 56 is 70% of what number?

4. What percent is 13 of 85? (Another way to ask this would be: What percent of 85 is 13?)

5. What is 16.2% of 85?

6. A computer system is reduced $495 from the original price. What is the percent decrease?

7. What number is 62% of 1200?

8. 425 is 85% of what number?

9. 38 is what percent of 5700?

10. 1950 is what percent of 5000?

11. 210 is 250% of what number?

12. What number is $33\frac{1}{3}\%$ of 516?

13. 24% of 300 is what?

14. Find 320% of 60.

15. .9% of 2000 is what?

16. 30 is _____% of 50.

17. 25% of _____ is 10.

18. _____ is 20% of 50.

5.7 Miscellaneous Signed Numbers & Critical Thinking *Homework*

Please answer with clear sentences.

1. What is the total purchase price if 25 acres are purchased for $380 per acre?

2. How much is missing from the PTA's fund if $832 remains from the original $1,405?

3. How much did the burglar get if he took three envelopes containing $129, $3,827 and $45, respectively?

4. How much can the investor expect to get back if she is promised a 1,000 percent return on a $40 investment?

5. The average APS customer can expect her new monthly bill to total how much if the current average bill of $110 is increasing by 8 percent?

6. What percent of the rancher's property has been sold if he has 200 acres remaining from the original 1,000 acre property?

Simplify the following expressions.

7. $\frac{3}{4}-\frac{2}{9}$ 8. $\frac{3}{14}-\frac{2}{7}$ 9. $-\frac{2}{5}+\frac{1}{3}$ 10. $-12-(-3)-9$ 11. $-\frac{6}{7}-\left(-\frac{5}{4}\right)$

12. $100-98$ 13. $(-2)+(-8)$ 14. $(-4)+7$ 15. $(10)+(-2)$ 16. $8+(-6)$

17. $(-7)+7$ 18. $-20-10$ 19. $5-8$ 20. $-3-2$ 21. $\frac{-40}{(-2)(-5)}$

22. $3-(-4)$ 23. $-4-1$ 24. $10\cdot 0$ 25. $4\cdot(-5)$ 26. $(-3)\cdot(-5)$

27. $(-1)^4$ 28. $(-2)(-1)(-3)$ 29. $\frac{-10}{-5}$ 30. $\frac{6}{-2}$ 31. $-1-(-9)$ 32. $\frac{-4}{12}$

We are what we repeatedly do. Excellence, therefore, is not an act but a habit.

Aristotle

Chapter 6 – Unit Conversions & Graphs

6.1 Unit Conversions Length—Area—Volume—Liquid—Weight—Temperature

Measuring Length English System - U.S. Metric System - Other Countries (Units are based on powers of ten)

Dimensional Analysis A procedure using unit fractions to convert from one unit of measurement to another.

Unit Fractions- Fractions where the numerator & denominator contain different units & the value of the unit fraction is 1. Example: $\frac{24\ hours}{1\ day} = 1$

USA System

			60 seconds	=	1 minute
			60 minutes	=	1 hour
12 inches.	= 1 foot		24 hours	=	1 day
36 inches	= 3 feet	= 1 yard	365 days	=	1 year
	5,280 ft.	= 1 mile	12 months	=	1 year

Example Convert 2 months to seconds. Convert 2 years to minutes.

6.1* Conversions *Homework: Use dimensional analysis to convert the quantity to the indicated unit rounded to two decimal places.

1) 100 in. to ft.	2) 100 ft. to in.	3) 21 in. to yd.	4) 21 yd. to in.
5) 12 yd. to ft.	6.) 12 ft to yd	7) 19,800 ft to mi	8) 0.25 mi to ft.

The Metric System

½ a mile

1 kilometer (km)	= 1000 meters (m)			
1 hectometer (hm)	= 100 meters			
1 dekameter (dam)	= 10 meters			
1 meter				
1 decimeter (dm)	= 1/ 10 meter	i.e.	10 decimeters	= 1 meter
1 centimeter (cm)	= 1/100 meter		100 centimeters	= 1 meter
1 millimeter (mm)	= 1/1000 meter		1000 millimeters	= 1 meter

Think of

kilo	**hecto**	**deka**		**deci**	**centi**	**milli**
for 1000	**for 100**	**for 10**		**for 1/10**	**for 1/100**	**for 1/1000**
kilometer	**hectometer**	**dekameter**	**meter**	**decimeter**	**centimeter**	**millimeter**
km	**hm**	**dam**	**m**	**dm**	**cm**	**mm**

Converting length in the metric system:
Multiply by 10 for each step to the right →
← Divide by 10 for each step to the left

6.1* Conversions *Homework continued: Convert as directed.
9) 8 dam to m 10) 0.37 hm to m 11) 8.64 hm to cm 12) 0.00037 km to cm 13) 71 dm to km

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English and Metric Equivalents

1 in ≈ 2.54 cm
1 ft ≈ 30.48 cm
1 yd ≈ 0.9 m
1 mi ≈ 1.6 km

$\frac{5280\text{ ft}}{1\text{ mi}}$, $\frac{12\text{ in.}}{1\text{ ft}}$, and $\frac{2.54\text{ cm}}{1\text{ in.}}$

Unit fraction $\frac{1\ inch}{2.54\ cm} = 1$

100 miles per hour in feet per second

$\frac{36\text{ in.}}{1\text{ yd}}$ and $\frac{2.54\text{ cm}}{1\text{ in.}} = 1$

6.1* Conversions *Homework continued: Convert as directed.
14) 26 in to cm 15) 26 cm to in 16) 776 mi to km 17) 776 km to mi
18) 20 m to yd 19) 1.2 dam to in 20) 180 in to hm 21) 8 ft to m
22) 8 m to ft 23) 104 kilometers per hour in miles per hour
24) 50 miles per hour in kilometers per hour 25) 8 yd to cm 26) 1016 cm to yd
27) 50 mi to km 28) 100 miles per hour in feet per second

Measuring Area

Area Measures in the USA

Square Units of Measure:

1 Square foot (ft^2) = 144 Square inches (in^2)

1 Square yard (yd^2) = 9 Square feet (ft^2)

1 Acre (a) = 43,560 ft^2 or 4840 yd^2

1 Square mile (mi^2) = 640 Acres

6.1* Conversions *Homework continued: Convert as directed.

29) 20m^2 to ft^2 30) 14 mi^2 to km^2 31) 20.6 ha to acres 32) 20 in^2 to cm^2

Measuring Volume

Cubic Units		**USA Capacity**
1 Cubic yard (yd^3)	≈	200 Gallons
1 Cubic foot (ft^3)	≈	7.48 Gallons
231 Cubic inches (in^3)	≈	1 Gallon

Liquid and Volume Measures in the USA (ounces, pounds, tons)

USA Capacity

1 Cup (c)	=	8 Ounces
2 Pints (pt)	=	1 Quart (qt)
4 Quarts	=	1 Gallon (gal)
1 Gallon	=	128 Ounces (oz)

Metric Liquid: Liter

Kilo **h**as **d**amian **L**ove **d**ollars **c**alled **m**illi.

kL	hL	daL	L	dL	cL	mL
kiloliter	**hectoliter**	**dekaliter**	**liter**	**deciliter**	**centiliter**	**milliliter**
1000L	100L	10L	1L	$\frac{1}{10}$L	$\frac{1}{100}$L	$\frac{1}{1000}$L
kg	hg	dag	g	dg	cg	mg
kilogram	**hectogram**	**dekagram**	**gram**	**decigram**	**centigram**	**milligram**
1000g	100g	10g	1g	$\frac{1}{10}g$	$\frac{1}{100}g$	$\frac{1}{1000}g$

Volume – Capacity – Weight in the Metric System

Metric Volume in Cubic Units	Liquid Capacity	Weight
$1\ cm^3$	= 1 mL	= 1 gram (g)
$1\ dm^3 = 1000\ cm^3$	= 1 L	= 1 kilogram (kg)
$1\ m^3$	= 1 kL	= 1000 kilograms (kg) = 1 Tonne (t)

Measuring Weight & Temperature

Weight Measure – English System (USA)

16 Ounces (oz) = 1 Pound (lb)
2000 Pounds (lb) = 1 Ton (T)

Weight Measure – USA and the Metric System

1 ounce (oz) = 28 Grams (g)
1 pound (lb) = 0.45 Kilograms (kg)
1 ton (T) = 0.9 Tonne (t)

Celsius to Fahrenheit:

$F = \frac{9}{5}(C) + 32$ $\frac{9}{5}\left(\frac{2}{7}\right)$

Fahrenheit to Celsius:

$C = \frac{5}{9}(F - 32)$ $\frac{9}{5} \cdot \frac{2}{7}$

6.1* Conversions *Homework continued: Convert as directed.

33) 25,000 ft^3 to gal 34) 6,237 in^3 to gal 35) 75,000 cm^3 to L 36) 4.5 L to cm^3

37) 16 kL to dm^3 38) 6.9 dg to mg 39) 0.37kg to g 40) 0.02m^3 to kg

41) 0.03kl to g 42) 26oz to lb 43) 220kg to lb 44) 20°C

45) 98°C 46) 86°F 47) 90°F

Selecting from millimeter, meter, and kilometer, determine the best unit of measure to express the given length.

48. The length of a football field
49. The distance from New York City to Washington, D.C.
50. The length of a car
51. The altitude of an airplane
52. The width of a human foot

Select the best estimate for the measure of the given item.

53. The length of this page a. 2.5 mm b. 25 mm c. 250 mm
54. The length of a pair of pants a. 700 cm b. 70 cm c. 7 cm
55. The length of a rowboat a. 4 cm b. 4 dm c. 4 m d. 4 dam
56. The dimensions of a piece of typing paper
    a. 22 mm by 28 mm b. 22 cm by 28 cm c. 22 dm by 28 dm d. 22 m by 28 m
57. If you jog six times around a track that is 700 meters long, how many kilometers have you covered?
58. The distance from New York City to Los Angeles is 4690 kilometers. What is the distance in miles?

Summation $\sum$

This symbol: $\sum_i^n x_i$ means to add up all the x values.

The first number in the parentheses is the "x" and the second number is the "y".

Example So if we had these four pairs (-2,7), (4,5), (6,6), (7,10)

$\sum_i^n x_i$ =-2+4+6+7= 15 and $\sum_i^n y_i$ = 7+5+6+10=28.

n is the number of pairs. In the example above, n is 4.

Example
Use these values to do the summations: (0,1.1), (1,5.1), (2,9.3), (3,13.1), (4,16.9)

a) n = _____

b) $\sum_i^n x_i$ = _____

c) $\sum_i^n y_i$ = _____

d) $\sum_i^n x^2{}_i$ = _____ (this means square each x and then add all of the squares together)

e) $\sum_i^n x_i\ y_i$ = _____(this means multiply x times y and then add all the products)

The first step in the acquisition of wisdom is silence, the second listening, the thirdl memory, the fourth practice, the fifth teaching others.

Solomon Ibn Gabriol

HOMEWORK ANSWERS FOLLOW

1.1 Numbers *Homework Answers*

Identify the number as *Whole Number, Natural Number, Integer, Negative Number, Positive Decimal, Negative Decimal, Improper Fraction, Proper Fraction, Mixed Number*

1. 15 whole number, natural number
2. $8\frac{3}{4}$ mixed number
3. 0 whole number
4. 3.781 positive decimal
5. 83,001 whole number, natural number
6. - 8 negative number
7. $\frac{7}{16}$ proper fraction
8. $\frac{9}{5}$ improper fraction
10. 33.7 mixed number
11. –5 negative number
12. 457 whole number, natural number
13. $\frac{8}{5}$ improper fraction
14. $1\frac{3}{4}$ mixed number
15. –14.1 negative decimal
16. 5.8 positive decimal
17. $\frac{5}{4}$ improper fraction
18. $\frac{1}{10}$ proper fraction
19. 362,049 whole number, natural number
20. 0.1 positive decimal
21. $7\frac{7}{8}$ mixed number
22. 33.7 positive decimal
23. $\frac{8}{5}$ improper fraction
24. $1\frac{3}{4}$ mixed number
25. π pi ≈ 3.14 or $\frac{22}{7}$
26. 75,039 whole number, natural number
27. $\frac{1}{3}$ proper fraction
28. - 87 negative number
29. 6.49 positive fraction
30. –0.5 negative decimal
31. $2\frac{7}{10}$ mixed number
32. $\frac{15}{8}$ improper fraction
33. 14, 50, $\sqrt{36}$

1.2 Place Value *Homework Answers*

Identify the place value of the digit **2** in each number.

1. 61,284 the 2 is in the hundreds place
2. 82,110 thousands place
3. 284,100 hundred thousands place
4. 823,415 ten thousands place
5. 725,837,166 ten millions place
6. 44,265,199 hundred thousands
7. 253,045,701,000 hundred billion place
8. 823,000,419,567 ten hundred billion

Name the place value for each **0** in this number: 302,016,450,098,570

9. 1st zero is in ten trillions place
10. 4th zero is in hundred thousands place
11. 2nd zero is in hundred billions place
12. 5th zero is in the ones place
13. 3rd zero is in the one millions place

Name the place value for each 0 in this number: 810,704,069,809,035

14. 1st zero is in the one trillions place
15. 4th zero is in the ten thousands place
16. 2nd zero is in the ten billions place
17. 5th zero is in the hundreds place
18. 3rd zero is in the hundred millions place

Write each number in words.

19. 8421 eight thousand four hundred twenty-one
20. 1936 one thousand nine hundred thirty-six
21. 46,205 forty-six thousand two hundred five
22. 75,089 seventy-five thousand eighty-nine
23. 3,064,801 three million sixty-four thousand eight hundred one
24. 7,900,408 seven million nine hundred thousand four hundred eight
25. 46,805
26. 79,046
27. 5,600,082
28. 1,030,005
29. 571,900,000
30. 311,000,400
31. 72
32. $0, -\frac{12}{3}, 72, -65$
33. $0, \frac{2}{3}, -4\frac{3}{5}, -\frac{12}{3}, \sqrt{\frac{81}{4}}, 72, -65$
34. 0 and 72

1.3 Rounding Numbers *Homework Answers*

1) 50 2) 90 3) 660 4) 120 5) 16,500 6) 12,800 7) 823,400
8) 701,500 9) 38,000 10) 117,000 11) 13,000 12) 672,000 13) 5,254,000
14) 1,396,000 15) 9,008,000 16) 3,116,000

1.4 Adding *Homework Answers*

1) 69	2) −103	3) 914	4) −38	5) 158
6) 234	7) 465	8) −507	9) 750	10) 437
11) −564	12) 456	13) −745	14) −6002	15) −808
16) 7,744,814	17) 6,765,985	18) 2,874,884	19) −6,047,114	
20) 9,872,381	21) −4,278,923	22) 5,872,917	23) 6,378,293	